

A STUDY OF DYNAMICAL BEHAVIOR OF SPACE ENVIRONMENT

by

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## CHAPTER I

### INTRODUCTION

Knowledge of the space environment is a necessity for the design and operation of space vehicles, the space shuttle, and Spacelab. Complex physical characteristics of the space environment are essential in providing a systematic background for the future advancement of knowledge.

In this report, we shall present the results of our studies during the period March 1970 - February 1974, of this contract. We have covered a wide range of problems in the space environment, such as the problems of the dynamical behavior of the thermosphere, hydromagnetic wave propagation in the ionosphere, and interplanetary space environment. The theories we used to analyze these problems range from a continuum theory of magnetohydrodynamics to the kinetic theory of free molecular flow. This is because the problems which we encountered cover the entire range of the Knudsen number (i.e., the ratio of mean free path to the characteristic length). From these studies, significant results were obtained. They are summarized as follows.

In the study of the dynamical behavior of the thermosphere, a theoretical model based on magnetohydrodynamic theory is suggested. We have shown that it is possible to predict the dynamical responses of the thermosphere due to a geomagnetic storm by using this model. It was demonstrated in this study that the quantities of wind speed, electric field strength, and joule heating due to a geomagnetic storm, which we predicted, agreed well with the observational data obtained by LOGACS (Low-G Accelerometer Calibration System) flown on a Agena satellite of the United States Air Force. Details of this investigation are given in Chapter II.

During the storm time, the ionospheric heating is a very important problem, which may trigger hydromagnetic waves. However, study of the propagation of hydromagnetic waves in the region of the ionosphere is a very complex problem due

to the variations of the collision frequencies of ions, electrons, and neutrals. Therefore, we shall restrict ourselves to dealing only with the propagation of hydromagnetic waves in the upper F2 region of the ionosphere. In this region, we find that electrons are in a transitional regime from collisional to collisionless conditions and ions are in a collisionless regime. Thus, we have derived a set of governing equations which are based on the fact that the isotropic electrons are fluid like, and the anisotropic ions follow kinetic equations modified by ion-electron collisions. It has been shown that the magneto-acoustic waves of a period of  $\sim 10$  seconds are dissipated by ion Landau damping and electron thermal conduction and viscosity. The numerical solutions under ionospheric conditions show that electron heating has always prevailed over ion heating in agreement with observations in the present study. The implications of possible mirror and/or firehose instabilities, which may cause turbulent or irregular fluctuations of fields, are discussed. The details of this study are included in Chapter III.

In Chapter IV we investigated the properties of the interplanetary medium, particularly that of the solar wind interaction with planetary objects. Because of the physical nature of the interplanetary medium (i.e., very large Knudsen number) a purely kinetic treatment is used for this study concerning the interaction of the solar wind with any "small" planetary object. "Small" refers to the cases where the thermal gyroradius of protons in the solar wind is arbitrarily taken to be  $\geq 0.1$  times the radius of the object under investigation. The "object" may possibly include an ionosphere or magnetosphere. The collisionless Boltzmann equation, neglecting the magnetic field, is used to calculate steady-state profiles of density and velocity around the object. In order to include the effects of surface reflection from the object, a finite, angularly distributed potential field on the surface of the object is also considered. A symmetric potential field is also examined for the purpose of comparison. A low density plasma void in the umbral region and a compression (above the value of the upstream undisturbed density) in the penumbral region are clearly found. The present technique, despite its neglect of the interplanetary magnetic field, is proposed as an alternative zeroth order approach to the continuum, local magnetic anomaly, and griding center approaches

used by others for the particular case of the moon. Some recent, potentially relevant observations on, and in front of, the moon are discussed. Of particular interest is the development of a high density gradient which emanates from the limit and is inclined in the direction of what is known in continuum theory as the "Mach cone." This structure leads to the following speculation: if the object's size were to increase (or, alternately, the proton gyroradius were to decrease) by one or two orders of magnitude, a shock wave would develop first at the limb, gradually building up to a conventional detached bow shock. Thus, gasdynamic theory - as we think of it in the Earth's case - can be constructed as required, on the basis of kinetic theory.

Finally, we shall outline a recommendation for future study in Chapter V.

## CHAPTER II

### DYNAMICS OF THERMOSPHERE

#### II-1 Introduction

A geomagnetic storm is a natural disturbance of the geomagnetic field on and above the earth's surface. Its origin and associated phenomena are very complicated. Theoretically, a variation of the magnetic field may be caused by the following phenomena: 1) variation of the geomagnetic poles, 2) presence of a local inhomogeneous electric field  $\frac{\partial \vec{B}}{\partial t} = - \nabla \times \vec{E}$ , and 3) variation of electric currents.

The magnetic field generated by the earth's magnetic poles is relatively steady. Its variation is negligible in the period of a geomagnetic storm. On the surface of the earth, the effect of the ionization of the atmosphere is small and there is no significant electrical field that can be detected. Therefore, it is believed that the geomagnetic disturbance recorded on the earth's surface is mainly due to the existence and variation of electric current flowing above the earth (Chapman and Bartel [1]). Indeed, if the electric current data were given for the whole environment of the earth, then the magnetic field could be calculated everywhere, at least theoretically. This would explain the disturbance of the magnetic field recorded on the earth's surface during the geomagnetic storm. Actually, there are always electric currents flowing above the earth, and their intensities and directions are changing from time to time. As a consequence, the geomagnetic field values recorded on the earth will never be constant. If the transient geomagnetic field variations are smooth and regular, we say it is quiet; otherwise, it is said to be disturbed.

On a quiet day, the magnetic variation proceeds mainly according to local solar time with a small part related to the moon. The two parts are called solar daily and lunar daily magnetic variations, and their correspondence to magnetic fields is denoted by Sq and L. During a magnetic disturbance, additional electric currents flow in the ionosphere. They are superimposed on the Sq and L currents. From the analysis of geomagnetic disturbance data, we see that at least five components of electric currents are involved in a magnetic storm. They are described as follows:



1. DCF = disturbance due to current attributable to solar corpuscular flux at magnetopause,
2. DR = disturbance due to magnetic ring current,
3. DP = polar current, which is strongest in auroral electrojets,
4. DT = disturbance due to magnetospheric tail currents, and
5. DG = disturbance due to induced ground currents.

Since the electric currents are flowing in the slightly ionized gas medium of the upper atmosphere, the other associated physical and dynamic effects such as wind generation, joule heating, and density and temperature variations can be predicted by magnetogasdynamic theory (Piddington [2]). A theoretical feature of the geomagnetic storm and its associated phenomena are illustrated in Diagram I. (See also [2], P. 13.)

Therefore, a complete analysis of the problem of the dynamic structure of the upper atmosphere due to a magnetic storm has to include consideration of the disturbance currents and their associated electric and magnetic fields and the dynamic qualities simultaneously since they are interrelated with each other (see Diagram I). However, the solution of this problem is difficult due to the complexity of the mathematics. Many authors have studied one or a few particular effects separately and made assumptions about the other physical qualities rather arbitrarily; Cole [3] studied the joule heating of moving ionized gas on the assumption of steady and uniform electric and magnetic fields. He concluded that the joule heating effect is significant during a geomagnetic storm. The joule heating may be on the order of  $10^{-5} \text{ erg cm}^{-3} \text{ sec}^{-1}$  in the region of 100 to 200 km, and the wind as high as  $10^5 \text{ cm/sec}$ , if the assumed magnetic and electric field configurations are correct. Later, he [4] extended the results by including viscous effects. Maeda and Kato [5] have given an excellent review of the problems of electrodynamics of the ionosphere in which the problems of conductivity, wind and the dynamo theory, drift and its effect on the ionospheric formation, and the interaction between wind and electromagnetic field are discussed in detail. Thomas [6], and Thomas and Ching [7], applying a one-dimensional vertical model, reproduced the height profile and the mean time lag of the density disturbance by assuming that the

heat input due to a magnetic storm is given. Volland and Mayer [8] reanalyzed the same problem using a three-dimensional thermospheric model.

In this study, a theoretical model is established for calculating the joule heating and winds from the geomagnetic variations recorded at storm time. The mathematical formulation is based on magnetohydrodynamic theory (Chang, Wu, and Smith [9]). Faraday's law is employed for determining the electric field from the magnetic field disturbance data during the geomagnetic storm.

## II-2 Basics of Theoretical Model

### II-2-1 Basic Equations

The thermosphere may be considered as a continuous medium with a finite electric conductivity (Cowling [10]). The presence of electric and magnetic fields in the conducting medium will give rise to two principal effects: First, body force (Lorentz force) and, second, energy generation (Joule heating). These must be taken into consideration in the momentum and energy equations. We will derive the equations of mass, momentum, and energy conservation as follows:

#### Mass Conservation

The equation of mass conservation is the same as in ordinary fluid dynamics, namely

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \quad (1)$$

where  $\rho$  denotes the mass density and  $\vec{v}$  the velocity vector.

#### Momentum Conservation

The equation of motion of a continuum medium in general can be written as (Cauchy equation)

$$\rho \frac{D v_i}{D t} = \frac{\partial P_{ij}}{\partial x_j} + F_i \quad (2)$$

where  $v_i$  are the components of the velocity vector  $\vec{v}$ ,  $P_{ij}$  are components of the stress tensor, and  $F_i$  denotes the components of the body force  $\vec{F}$ . For a Newtonian fluid the stress tensor can be expressed as follows:

$$P_{ij} = -p + \frac{2}{3} \eta (\nabla \cdot \vec{v}) \delta_{ij} - \eta \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) \quad (3)$$

where  $p$  is the static pressure and  $\eta$  is the viscosity. By substituting Eq. (2) into Eq. (3) we obtain

$$\rho \frac{D \vec{v}}{D t} = - \nabla p + \vec{\chi} + \vec{F} \quad (4)$$

where

$$\vec{\chi} = -\frac{2}{3} \nabla (\eta \nabla \cdot \vec{v}) + \eta [\nabla^2 \vec{v} + \nabla (\nabla \cdot \vec{v})] \\ + 2 [(\nabla \eta) \cdot \nabla) \vec{v} + (\nabla \eta) \times (\nabla \times \vec{v})]$$

and the body force  $\vec{F}$  is

$$\vec{F} = \rho_e \vec{E} + \vec{j} \times \vec{B} + \rho \vec{g}$$

where  $\rho_e$  is the charge density,  $\vec{E}$  is the electric field strength,  $\vec{B}$  is the magnetic induction, and  $\vec{j}$  is the sum of the conduction current and the current flow due to convective transport of charges.

### Energy Conservation

The rate of increase of total energy in the fluid of a moving volume  $\sigma$  is given by

$$\int \frac{D\phi}{Dt} d\sigma = \int \frac{1}{2} \frac{D(\rho v^2)}{Dt} d\sigma + \int \frac{D(\rho e)}{Dt} d\sigma$$

where  $e$  is the internal energy per unit mass.

In order for energy to be conserved, this must equal the energy inputs per unit of time from other sources. These are

1. Joule heating =  $\int (\vec{E} \cdot \vec{j}) d\sigma$
2. Heat conduction per unit time =  $-\int \nabla \cdot (\lambda \nabla T) d\sigma$ , where  $\lambda$  is the thermal conductivity.
3. Work done by surface force =  $-\int \sum_{i,j} v_i P_{ij} dS_j$  applying Gauss' theorem

$$-\int_S \sum_{i,j} v_i P_{ij} dS_j = -\int_V \sum_{i,j} \frac{\partial}{\partial x_j} (v_i P_{ij}) d\sigma$$

The energy equation is obtained as

$$\int_V \frac{D\phi}{Dt} d\sigma = \int_V \vec{E} \cdot \vec{j} d\sigma - \int_V \nabla \cdot (\lambda \nabla T) d\sigma \\ - \int_V \sum_{i,j} \frac{\partial}{\partial x_j} (v_i P_{ij}) d\sigma \quad (5)$$

The last summation on the right hand side of Eq. (5) can be rewritten as

$$- \sum_i \frac{\partial}{\partial x_i} (p v_i) + \varphi$$

where, from Eq. (3)

$$\begin{aligned} \varphi = & \eta \sum_i v_i \nabla^2 v_i + \frac{1}{3} \eta \sum_i v_i \frac{\partial}{\partial x_i} \left( \sum_j \frac{\partial v_j}{\partial x_j} \right) \\ & + \sum_{ij} v_i \frac{\partial \eta}{\partial x_j} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) + \frac{2}{3} \sum_i v_i \frac{\partial \eta}{\partial x_i} \left( \sum_j \frac{\partial v_j}{\partial x_j} \right) \\ & + \eta \sum_{ij} \frac{\partial v_i}{\partial x_j} \left( \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) - \frac{2}{3} \eta \sum_j \left( \frac{\partial v_j}{\partial x_j} \right)^2 \end{aligned}$$

### Maxwell Equations and their Approximation

The Maxwell equations are needed to determine the electromagnetic quantities. Thus,

$$\nabla \cdot \vec{B} = 0 \quad (6)$$

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon} \rho_e \quad (7)$$

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (8)$$

$$\nabla \times \vec{B} = \mu_o \left( \vec{J} + \epsilon \frac{\partial \vec{E}}{\partial t} \right) \quad (9)$$

and Ohm's Law;  $\vec{J} = \sigma (\vec{E} + \vec{v} \times \vec{B}) + \sigma_2 \left[ \vec{B} \times (\vec{E} + \vec{v} \times \vec{B}) \right] / |\vec{B}| \quad (10)$

A number of approximations can be made which will be valid as far as the present analysis is concerned:

1) The displacement current  $\epsilon \frac{\partial \vec{E}}{\partial t}$  may be neglected in Eq. (9). In order to show that this is a valid approximation, we shall compare  $\epsilon \frac{\partial \vec{E}}{\partial t}$  to the conduction current  $\vec{J} \approx \sigma \vec{E}$ . If  $\vec{E}$  is assumed to vary periodically with time with frequency  $\omega$ , then the ratio of the amplitude of  $\epsilon \frac{\partial \vec{E}}{\partial t}$  to the amplitude of  $\vec{J}$  is approximately

$$\frac{\epsilon (\partial \vec{E} / \partial t)_{\max}}{(\sigma \vec{E})_{\max}} = \frac{\epsilon \omega}{\sigma} \quad \frac{\epsilon \left( \frac{\partial \vec{E}}{\partial t} \right)_{\max}}{(\sigma \vec{E})_{\max}} = \frac{\epsilon \omega}{\sigma}$$

The value for  $\sigma$  in our case is about  $10^{-3}$  mhos/m. The value of  $\epsilon$  in a vacuum is approximately  $9 \times 10^{-12}$  farads/m.  $\omega$  is about  $10^{-2} \text{ sec}^{-1}$ . This gives

$$\frac{\epsilon (\partial \vec{E} / \partial t)_{\max}}{(\sigma \vec{E})_{\max}} \approx 10^{-10}$$

which shows that the displacement current can be indeed neglected.

2) The electrostatic body force can be neglected.

If  $\vec{E}$  varies linearly over some small region, then from Eq. (7) we see

$$\rho_e \approx \frac{\epsilon E}{L}$$

Therefore

$$\rho_e E \approx \frac{\epsilon E^2}{L}$$

In comparison to the Lorentzian force  $\vec{J} \times \vec{B} \approx \sigma v B^2$ , we have

$$\frac{\rho_e E}{\vec{J} \times \vec{B}} \approx \frac{\epsilon E^2}{\sigma L v B^2} \approx \frac{\epsilon v^2 B^2}{\sigma L v B^2} = \frac{\epsilon v}{\sigma L}$$

The values in our problem are estimated as follows

$$\epsilon \approx 10^{-11} \text{ farads/m}$$

$$v \approx 10^3 \text{ m/sec}$$

$$L \approx 10^6 \text{ m}$$

$$\sigma \approx 10^{-3} \text{ mhos/m}$$

$$\frac{\epsilon v}{L \sigma} \approx 10^{-11}$$

Therefore, the electrostatic body force can be neglected in comparison to the Lorentz force.

Now, let us recollect the above derived equations as follows:

Maxwell's equations:

$$\nabla \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (11)$$

$$\nabla \times \vec{B} = \mu_0 \vec{j} \quad (12)$$

$$\nabla \cdot \vec{B} = 0 \quad (13)$$

$$\epsilon \nabla \cdot \vec{E} = \rho_e \quad (14)$$

Conservation of Mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \quad (15)$$

Equation of Motion:

$$\rho \frac{D \vec{v}}{Dt} = - \nabla p + \vec{\chi} + \vec{j} \times \vec{B} \quad (16)$$

Conservation of Energy:

$$\rho \frac{D \phi}{Dt} = \vec{E} \cdot \vec{j} + \nabla \cdot (\lambda \nabla T) - \nabla \cdot (p \vec{v}) + \varphi \quad (17)$$

Ohm's Law:

$$\vec{j} = \sigma_1 \vec{E}' + \sigma_2 \frac{\vec{B} \times \vec{E}'}{|\vec{B}|} \quad (18)$$

where  $\vec{E}' = \vec{E} + \vec{v} \times \vec{B}$ . This set of equations will form the basis of our theoretical analysis. Various approximations will be introduced.

## II-2-2 Some Fundamental Solutions

Since the magnetohydrodynamic equations [Eqs. (11) - (18)] combine the full complexity of Maxwell's equations and the fluid dynamic equations, it is obvious that they will be extremely difficult to solve in the general form. Exact solutions exist only for a few special cases. We will discuss two of them; namely, the Piddington [11] solution for a non-viscous gas and the Hartman solution for a viscous fluid [12]. They do not necessarily correspond to the conditions during magnetic storm; however, the solutions will give us some insight into the physical nature of the problem.

(A) Consider a uniform ionized gas in steady and homogeneous electric and magnetic fields. They are assumed to have the following forms

$$\vec{E} = \vec{E} (E_x, E_y, 0) \text{ and } \vec{B} = (0, 0, B_z) .$$

We further assume that the pressure gradient is

$$\vec{\nabla} p = \vec{i} \frac{\partial p}{\partial x}, \text{ where } \vec{i} \text{ denotes the unit vector in the } x\text{-direction.}$$

By applying Eq. (18) and Eq. (16) and neglecting the viscous term, we obtain

$$\vec{j} = \sigma_1 \vec{E}' + \sigma_2 \frac{\vec{B} \times \vec{E}'}{B_z} \quad (19)$$

and

$$\rho \frac{D\vec{v}}{Dt} = \vec{j} \times \vec{B} - \nabla p \quad (20)$$

At hydrostatic equilibrium  $\frac{D\vec{v}}{Dt} = 0$ , then

$$\vec{j} = \frac{-\nabla p \times \vec{B}}{B_z^2} = \sigma_1 \vec{E}' + \sigma_2 \frac{\vec{B} \times \vec{E}'}{B_z} .$$



It follows that

$$v_x = E_x / B_z - \frac{\partial p}{\partial x} \frac{1}{B_z^2 \sigma_3} ; v_y = -E_x / B_z + \frac{\partial p}{\partial x} \frac{\sigma_2}{B_z^2 \sigma_1 \sigma_3} \quad (21)$$

$$j_x = 0 , \quad j_y = \frac{\partial p}{\partial x} \frac{1}{B_z} \quad (22)$$

where  $\sigma_3 = \sigma_1 + \sigma_2^2 / \sigma_1$  .

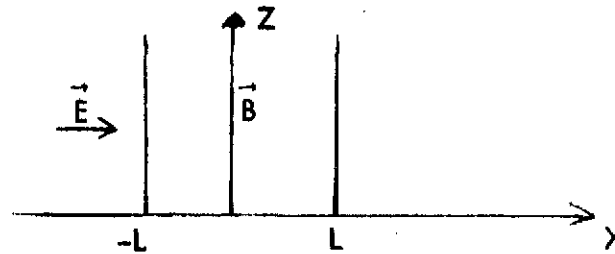
Therefore, the current  $\vec{j}$  and velocity  $\vec{v}$  are obtained in terms of  $\vec{E}$ ,  $\vec{B}$  and  $\nabla p$  . The Joule heating is given by

$$Q = \vec{j} \cdot \vec{E}' = \vec{j} \cdot (\vec{E} + \vec{v} \times \vec{B}) \quad (23)$$

This means that if we simulate the mechanism of geomagnetic activity by switching on the electric field  $\vec{E}$  , after a period of time the velocity  $\vec{v}$  and Joule heating  $Q$  will reach the values given by Eq. (21) and Eq. (23).

#### (B) Hartmann Solution

Cole [ 4 ] studied the problem of heating and dynamics near auroral electrojets by considering the following model as shown below:



The directions of the electric and magnetic fields are shown in the figure and the direction of flow is perpendicular to the  $x - z$  plane . If the viscous effect of the fluid is taken into consideration, the momentum equation can be written as:

$$\eta \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_y}{\partial z^2} - j_x B = 0 \quad (24)$$

where  $j_x = \sigma_1 (E + v_y B_z)$ .

Equation (24) can be rewritten as

$$\frac{\partial^2 v_y}{\partial x^2} + \frac{\partial^2 v_y}{\partial z^2} - \frac{\sigma_1}{\mu} |\vec{B}|^2 v_y = \frac{\sigma_2 |\vec{E}| |\vec{B}|}{\eta} \quad (25)$$

Let us consider the thickness of the sheet of the auroral zone to be  $2L$  and assume that the motion of the gas outside this sheet is small. We can then set the boundary condition as  $v_y = 0$  at  $x = \pm L$ . Then the solution of Eq. (25) that satisfies this boundary condition is given by

$$v_y(x) = \frac{|\vec{E}|}{|\vec{B}|} \left( 1 - \frac{\cosh(M_1 x/L)}{\cosh M_1} \right) \quad (26)$$

where  $M_1 = |\vec{B}| L (\sigma_1/\mu)^{1/2}$ . The heating within the sheet can be expressed as follows:

$$Q = \eta \left( \frac{|\vec{E}|}{|\vec{B}|} \frac{M_1}{L} \right)^2 \frac{\cosh(M_1 x/L)}{\cosh M_1} \quad (27)$$

### II-2-3 Time Dependent Problem

In the last section we have discussed steady solutions in magnetogasdynamics which indeed can help to understand the basic mechanism of Joule heating during a geomagnetic storm. However, they are not sufficient to explain the time variation of the geomagnetic disturbance. Actually all the quantities—electric field, magnetic field and velocity—are time dependent during the entire period of a geomagnetic storm. Therefore, the analysis for the unsteady case is necessary.

Let us recall the equation of motion, namely Eq. (16)

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \vec{\chi} + \vec{j} \times \vec{B}$$

Suppose that the current data  $\vec{j}$  are known (by either observation or theory). Then the induced magnetic field may be calculated by Eq. (8), together with the earth's

magnetic field. Therefore, the last term of Eq. (16) becomes known. The problem is then reduced to a dynamic problem of the atmosphere with an additional given driving force (Lorentz force) and heat source (Joule heating).

On the other hand, if the magnetic field data in the thermosphere are available, we will simplify the problem by eliminating the variables  $\vec{E}$  and  $\vec{j}$  from the system of equations. This is done by solving Eq. (18), namely,

$$\vec{E} = \frac{\vec{j}}{\sigma_1} - \vec{v} \times \vec{B} + \alpha (\vec{j} \times \vec{B}) \quad (28)$$

where  $\alpha = \sigma_2 / \sigma_1^2 |\vec{B}|$ .

Substitution of  $\vec{j}$  from Eq. (12) then gives

$$\vec{E} = \frac{\nabla \times \vec{B}}{\mu_0 \sigma_1} - \vec{v} \times \vec{B} + \alpha \frac{\nabla \times \vec{B}}{\mu_0} \times \vec{B} \quad (29)$$

which, when combined with Eq. (11), gives

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \frac{\nabla \times \vec{B}}{\mu_0 \sigma_1} - \nabla \times (\vec{v} \times \vec{B}) + \nabla \times \frac{\alpha}{\mu_0} (\nabla \times \vec{B}) \times \vec{B} = 0 \quad (30)$$

We notice that Eq. (30) includes only two variables  $\vec{B}$  and  $\vec{v}$ . If  $\vec{B}(x, y, z, t)$  is assumed to be given, then the velocity vector can be solved from Eq. (30) with properly imposed boundary conditions.

Equations (11-18) may be reduced in the following form

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0 \quad (31)$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \vec{\chi} + \frac{(\nabla \times \vec{B}) \times \vec{B}}{\mu_0} \quad (32)$$

$$\rho \frac{D\Phi}{Dt} = \frac{(\nabla \times \vec{B})^2}{2\mu_0 \sigma} + \nabla \cdot (\lambda \nabla T) - \nabla \cdot (\rho \vec{v}) + \varphi \quad (33)$$

This set of equations together with Eq. (30) formed a complete description of the time dependent problem. As observed from these equations, the only electromagnetic quantity explicitly involved in this system of equations (30-33) is the magnetic field  $\vec{B}$ . If  $\vec{B}$  is given, this set of equations is reduced to ordinary dynamic equations.

## II-3 Disturbed Thermosphere; Dynamic Responses of Thermosphere

### II-3-1 Statement of the Problem

A simplified model considers the thermosphere (80-200 Km) as a neutral inviscid atmosphere with finite electric conductivity. Hall effect can be neglected as we have shown in Section II. Thus, the set of fundamental equations is as follows:

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad (34)$$

$$\frac{D\vec{v}}{Dt} = - \frac{1}{\rho} \vec{\nabla} p + \frac{1}{\rho} (\vec{j} \times \vec{B}) \quad (35)$$

$$\rho C_p \frac{DT}{Dt} = \text{div} (\lambda \vec{\nabla} T) + \vec{j} \cdot \vec{E}' \quad (36)$$

$$\vec{E}' = \vec{E} + \vec{v} \times \vec{B} \quad (37)$$

$$\vec{\nabla} \times \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (38)$$

$$\vec{j} = \sigma (\vec{E} + \vec{v} \times \vec{B}) \quad (39)$$

and

$$p = \rho R T \quad (40)$$

We further assume that the atmospheric motions are confined to the vertical (z - axis) and horizontal (x-axis, E-W) directions. For the magnetic field, since observational data are not available at this stage, we assume quite arbitrarily that

$$\vec{B}(x, y, t) = \vec{b}(t) \exp(-\beta r), \quad (41)$$

where  $r = \sqrt{x^2 + y^2}$  and  $\beta$  is an arbitrary constant, which is the measure of the damping rate of geomagnetic variations. This assumed expression for the disturbed geomagnetic variations is such that the maximum disturbance is at the center of the region in which we are interested, and it will decay exponentially according to the distance from the center, which also implies that the geomagnetic disturbances are confined in a finite region. Furthermore, the altitude dependence is also ignored in the present study. The quantity  $\vec{b}(t)$  is taken from ground station magnetic data. In this calculation the May 1967 geomagnetic storm data recorded at College, Alaska; Dallas, Texas; and Honolulu, Hawaii were used. The 1964 Jacchia model atmosphere was used to calculate the pre-storm conditions of the atmosphere. The set of equations (34-41) was integrated numerically by using a finite difference technique. The numerical process is explained in Diagram (2). It includes the following steps.

1. Calculate  $\vec{E}$  from Eqs. (38) and (41).
2. Obtain  $\vec{E}'$  from Eq. (37).
3. Calculate  $\vec{j}$  from Eq. (39).
4. Integrate Eq. (35), determine  $\vec{v}$  for next time step,  $\vec{v}(\Delta t)$ .
5. Integrate Eq. (36), calculate  $T(\Delta t)$ .
6. Calculate  $p(\Delta t)$  from Eq. (40).
7. Determine  $\rho(\Delta t)$  from Eq. (34).
8. Use  $\vec{v}(\Delta t)$  to calculate  $\vec{E}'(\Delta t)$  from Eq. (37).

### II-3-2 Results

Numerical results are presented for winds, joule heating and electric field at the center of the storm and an altitude of 140 Km May 1967. The calculations are based on the observational data recorded at College, Alaska; Dallas, Texas; and Honolulu, Hawaii. The Alaskan results show that the geomagnetic storm can generate the horizontal wind (East-West direction) (Figure II-4) on the order of maximum  $1000 \text{ M. sec.}^{-1}$  and vertical wind (Fig. II-5) approximately an order of magnitude smaller, which agree with the recent electrojet observation Fees [13]. The joule heating (Fig. II-6) is about a few  $\text{erg cm}^{-3}\text{-sec.}^{-1}$  and the electric field (Fig. II-3)

on the order of  $100 \text{ mV} \cdot \text{m}^{-1}$ , which agree well with the results given by Cole [14] and Wu, Matsushita, and De Vries [15]. However, the calculated temperature in the present model is rather high. We believe this is due to the fact that the present model has not taken into account the effects due to viscosity. Considerably smaller winds, joule heating, and electric field result when the Texas and the Hawaiian data are used instead of the College, Alaska data. The horizontal (East-West) wind  $v_x$  over Dallas, Texas (Fig. II-10) is calculated to be  $\sim 100 \text{ m} \cdot \text{sec}^{-1}$ , and the vertical wind  $v_z$  (Fig. 11) is  $\sim 10 \text{ m} \cdot \text{sec}^{-1}$ . The joule heating (Fig. II-12) is approximately  $10^{-2} \text{ erg} \cdot \text{m}^{-3} \cdot \text{sec}^{-1}$  and the electric field (Fig. II-9) is on the order of  $10 \text{ mV} \cdot \text{m}^{-1}$ . The Hawaiian results show that  $v \sim 10 \text{ m} \cdot \text{sec}^{-1}$  (Fig. 16),  $v_z \sim \text{m} \cdot \text{sec}^{-1}$  (Fig. II-17),  $Q \sim 10^{-4} \text{ erg} \cdot \text{m}^{-3} \cdot \text{sec}^{-1}$  (Fig. II-18) and  $E \sim 1 \text{ m} \cdot \text{sec}^{-1}$  (Fig. II-15). The results show that the strongest thermal and dynamic responses in the thermosphere due to the magnetic storm are in the polar region and they become weaker as latitude decreases.

## II-4 Enclosure

In this study, a theoretical model for the dynamical responses to the geomagnetic storm in the thermosphere is derived from magnetohydrodynamic theory. The validity of this proposed model is based on the assumption that the thermosphere behaves as an electric conducting fluid. This is true because the degree of ionization in the thermosphere is  $\sim 10^{-4}$  and thus the dynamical properties in the thermosphere are dominated by the neutral gas. In order to test the proposed model, a numerical example is presented. In this calculation, we have neglected the viscous effects and fixed our attention only on a narrow region of the thermosphere, and constant transport properties are assumed in the model. In general, the results we obtained are in good agreement with the wind measurements from an Agena satellite [13] and joule heating obtained by Cole [3]. However, the calculated density and temperature in this model are unreasonably high. We believe this is because we have ignored the viscous effects, the altitude dependence, and the gravitational wave effects. Therefore, we shall recommend that this model be improved in the following manner:

- 1) By including the viscous effect.
- 2) By including a global calculation model using a suitable spherical coordinate system. Thus, the latitude dependence can be incorporated into the model.
- 3) Currently, our results are obtained for a point. We hope to do a calculation to include whole region of the atmosphere from 90 Km - 250 Km. the structure of the winds and electric field can be obtained.
- 4) By examining joule heating and the effects on gravitational waves.

These will form a basis of our future studies.



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## MHD MODEL OF THERMOSPHERE

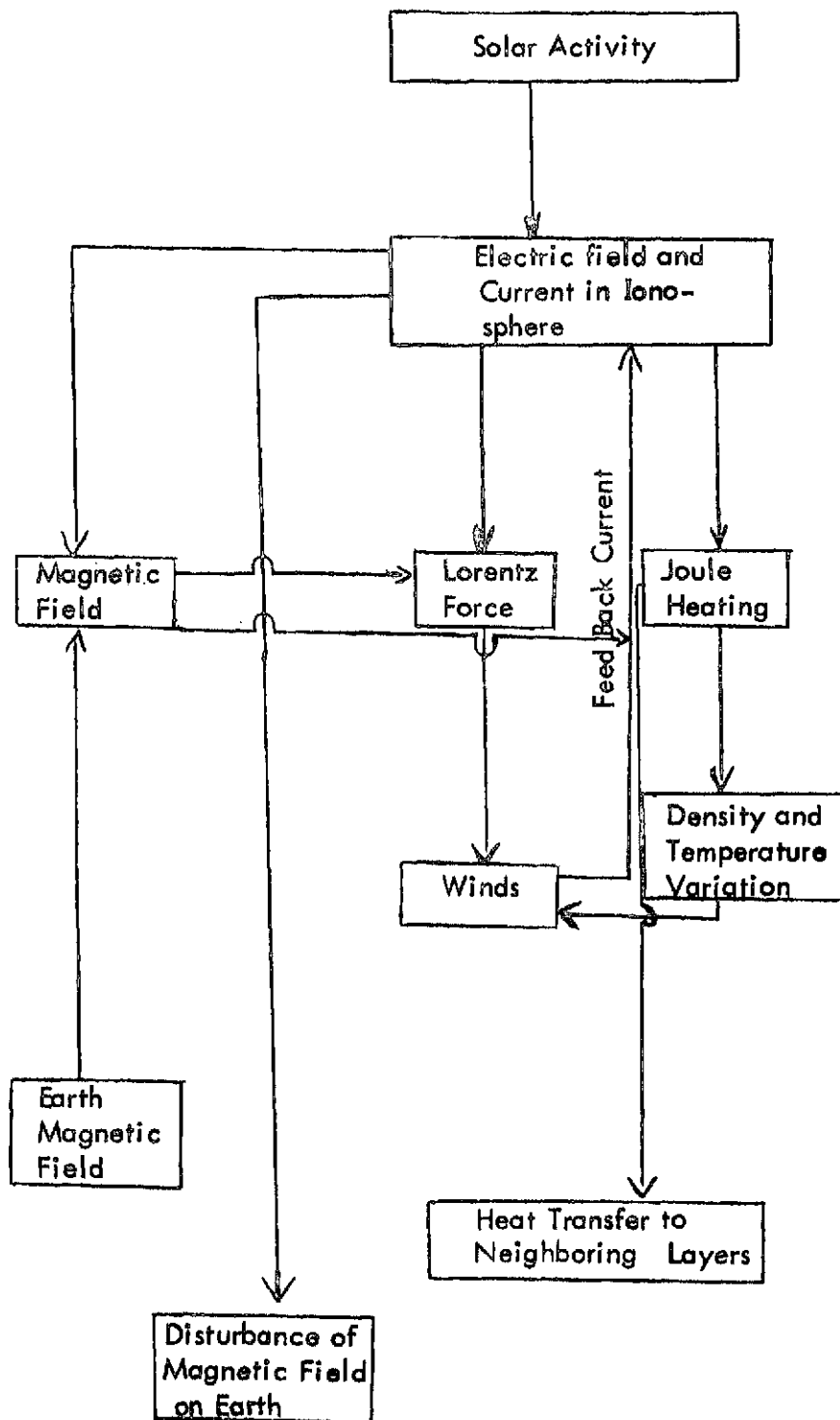


DIAGRAM I

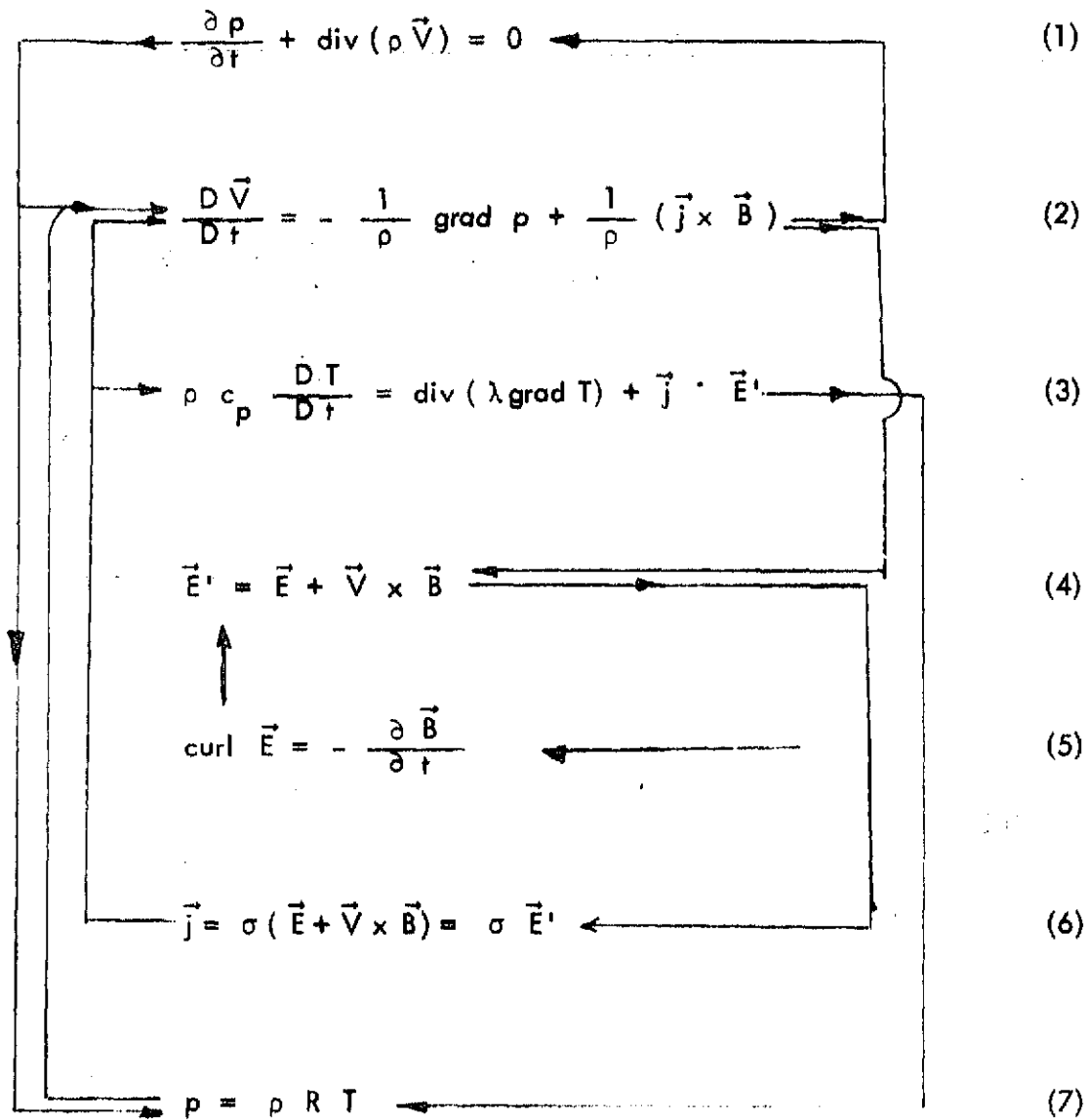


DIAGRAM II

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- Figure II-2 Perturbations in vertical component of magnetic field at College, Alaska for May 25-26, 1967.
- Figure II-3 Electric field at 140 km over College, Alaska on May 25-26, 1967.
- Figure II-4 Horizontal wind velocity at 140 km over College, Alaska on May 25-26, 1967.
- Figure II-5 Vertical wind velocity at 140 km over College, Alaska on May 25-26, 1967.
- Figure II-6 Joule heating at 140 km over College, Alaska on May 25-26, 1967.
- Figure II-7 Perturbation in horizontal component of magnetic field at Dallas, Texas for May 25-26, 1967.
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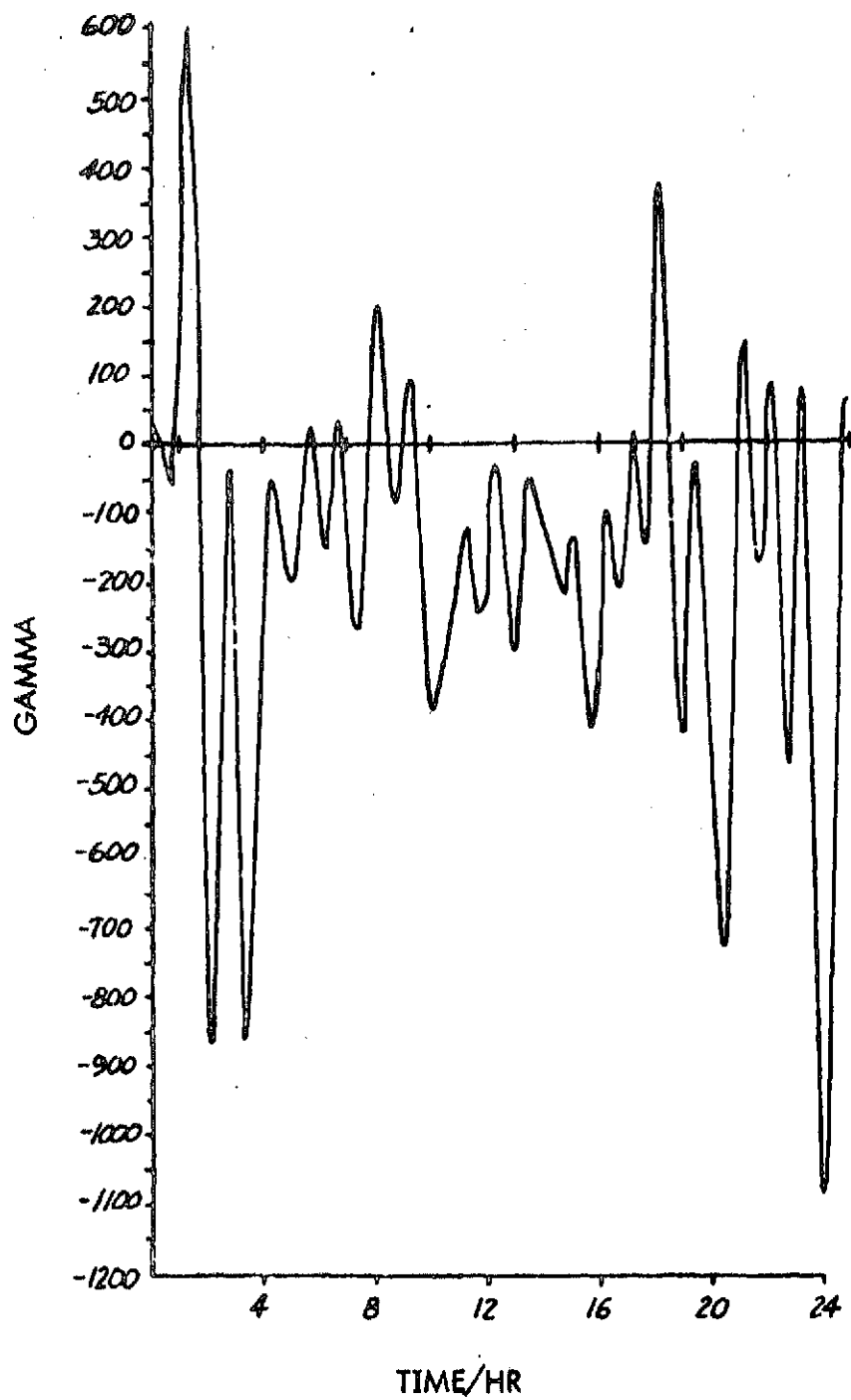


FIG. II-1

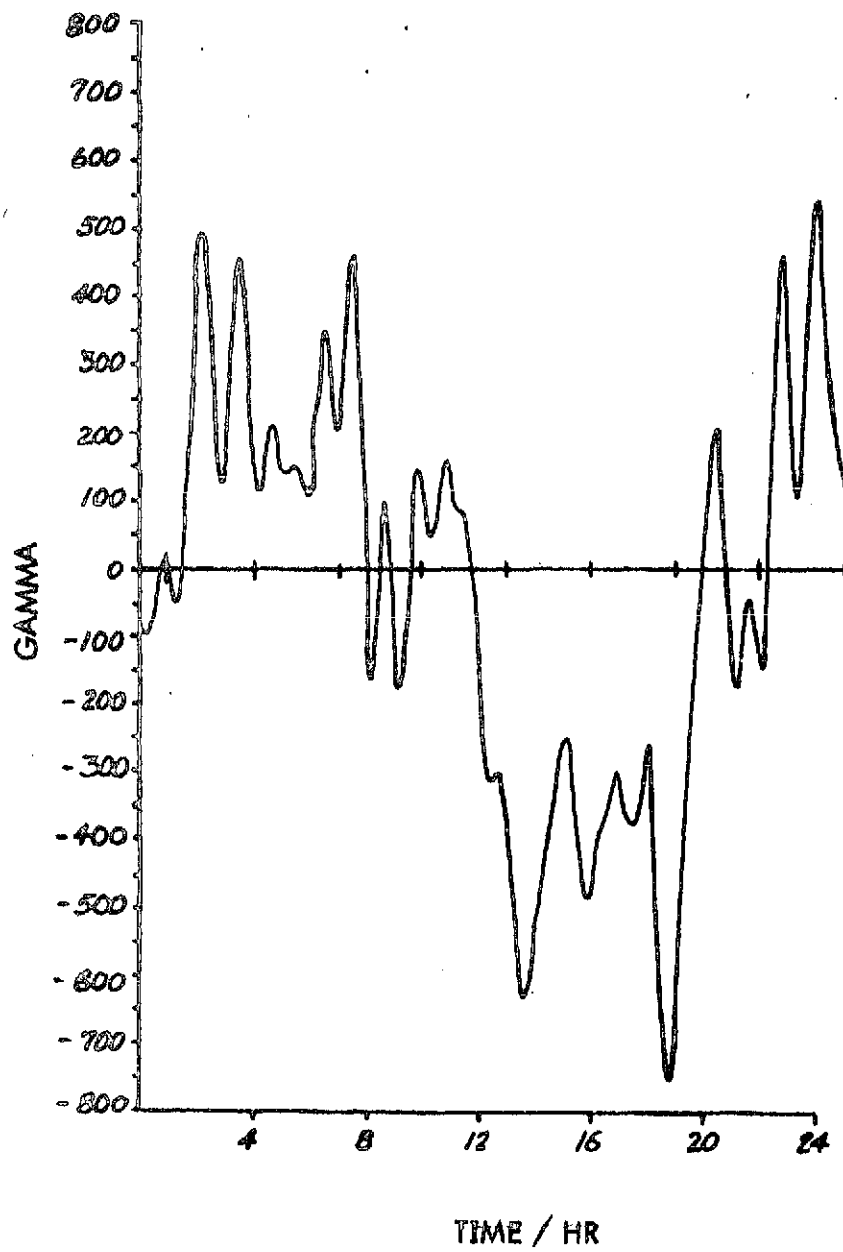


FIG. II-2

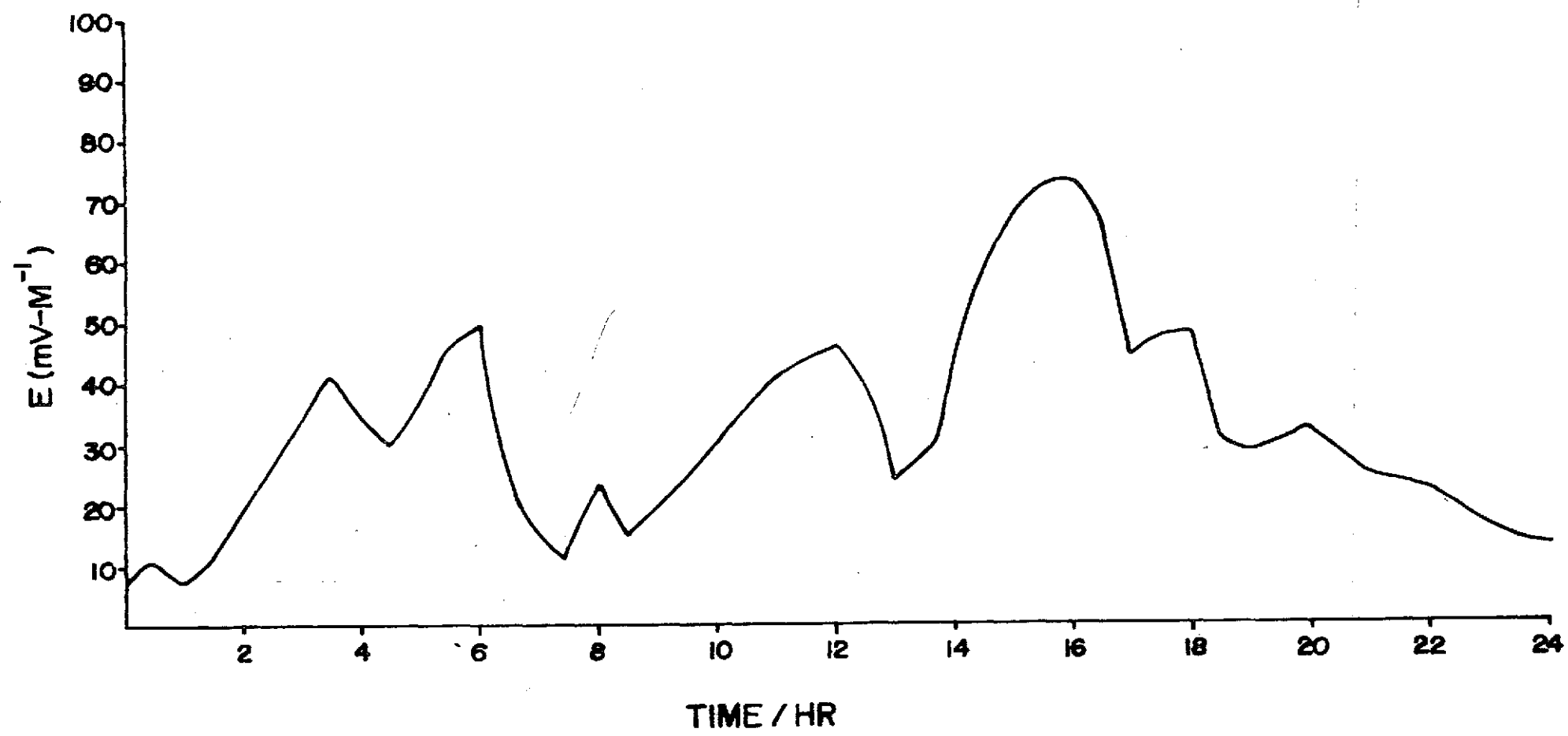


FIG. II-3

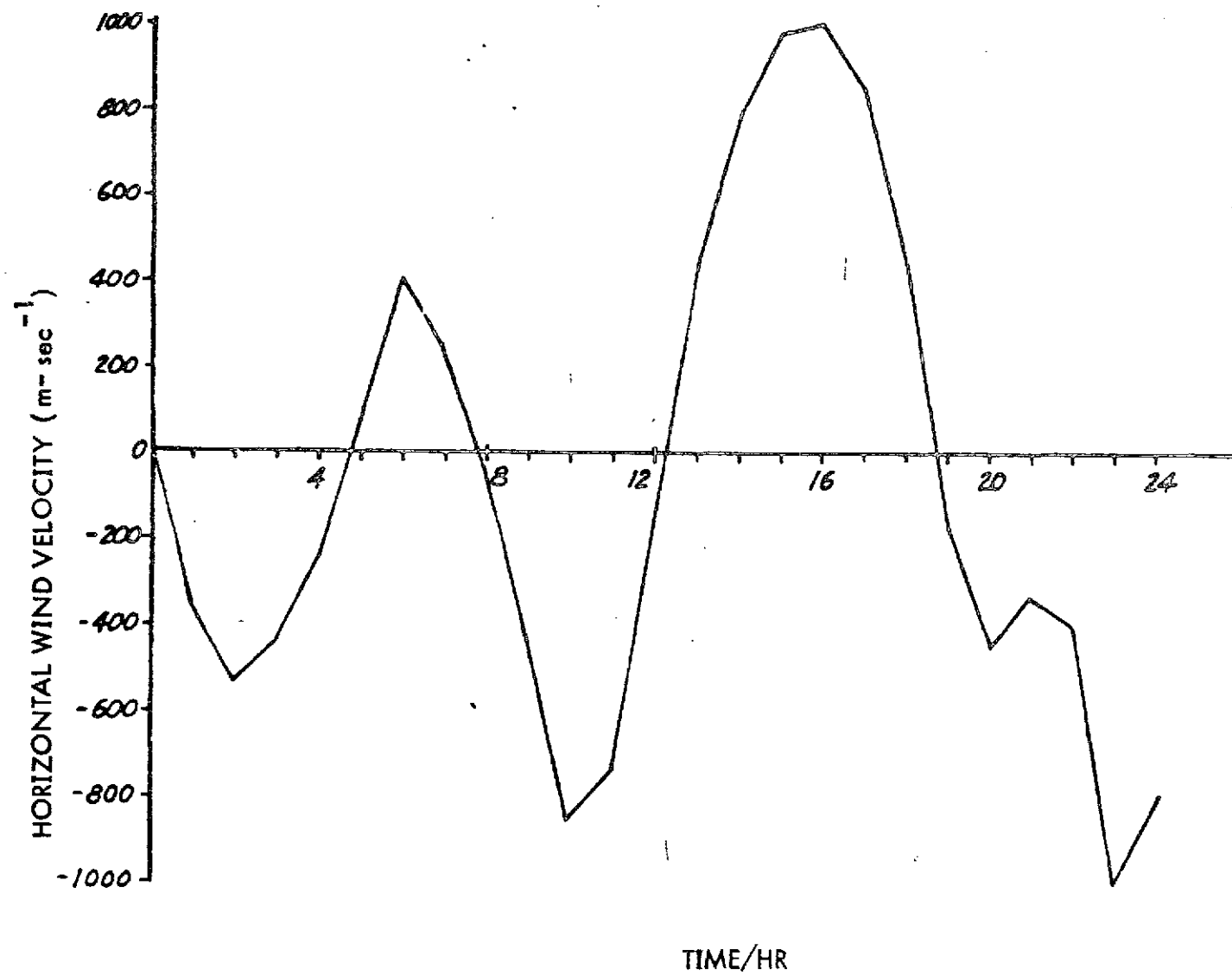


FIG. II-4



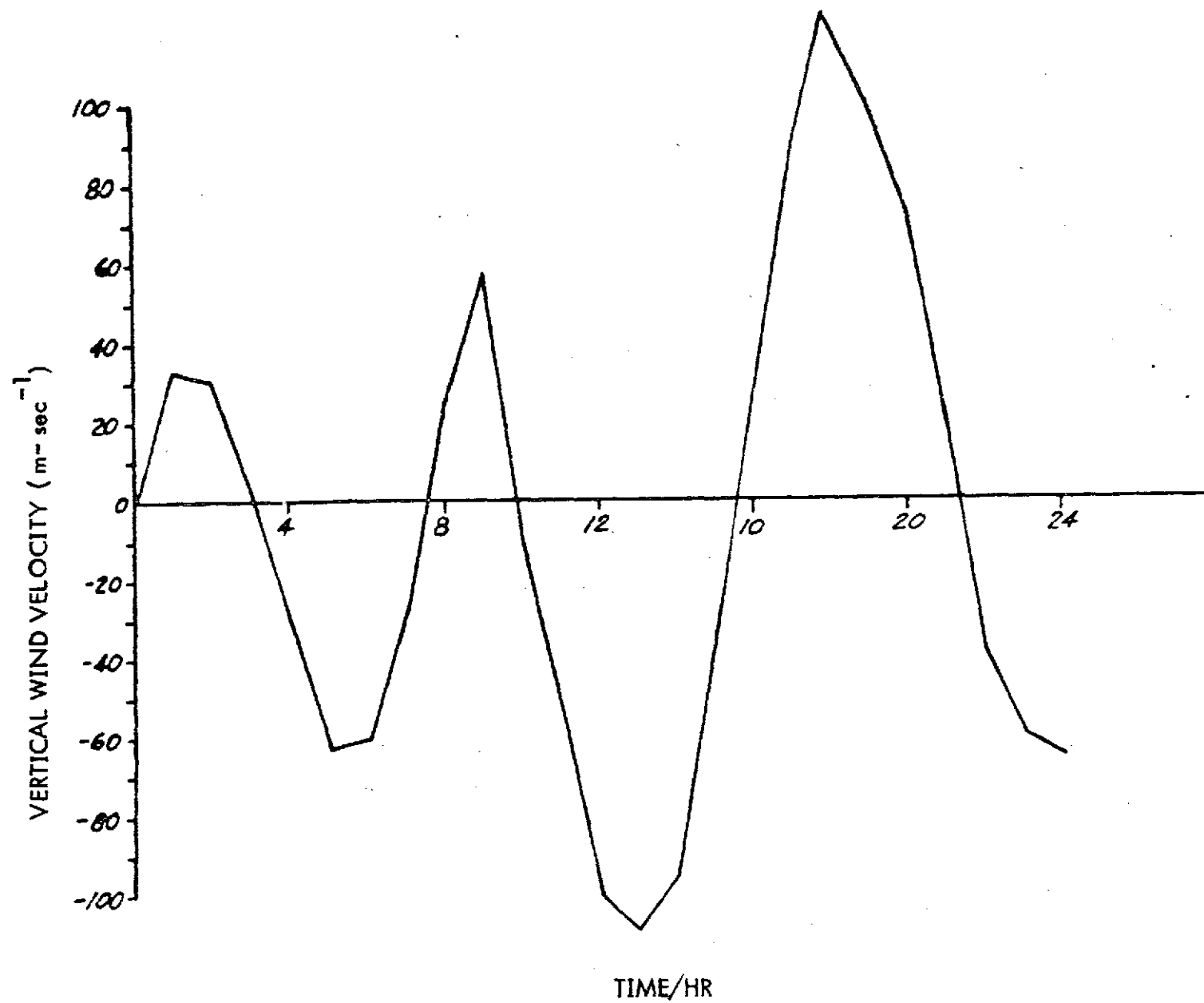


FIG. II-5

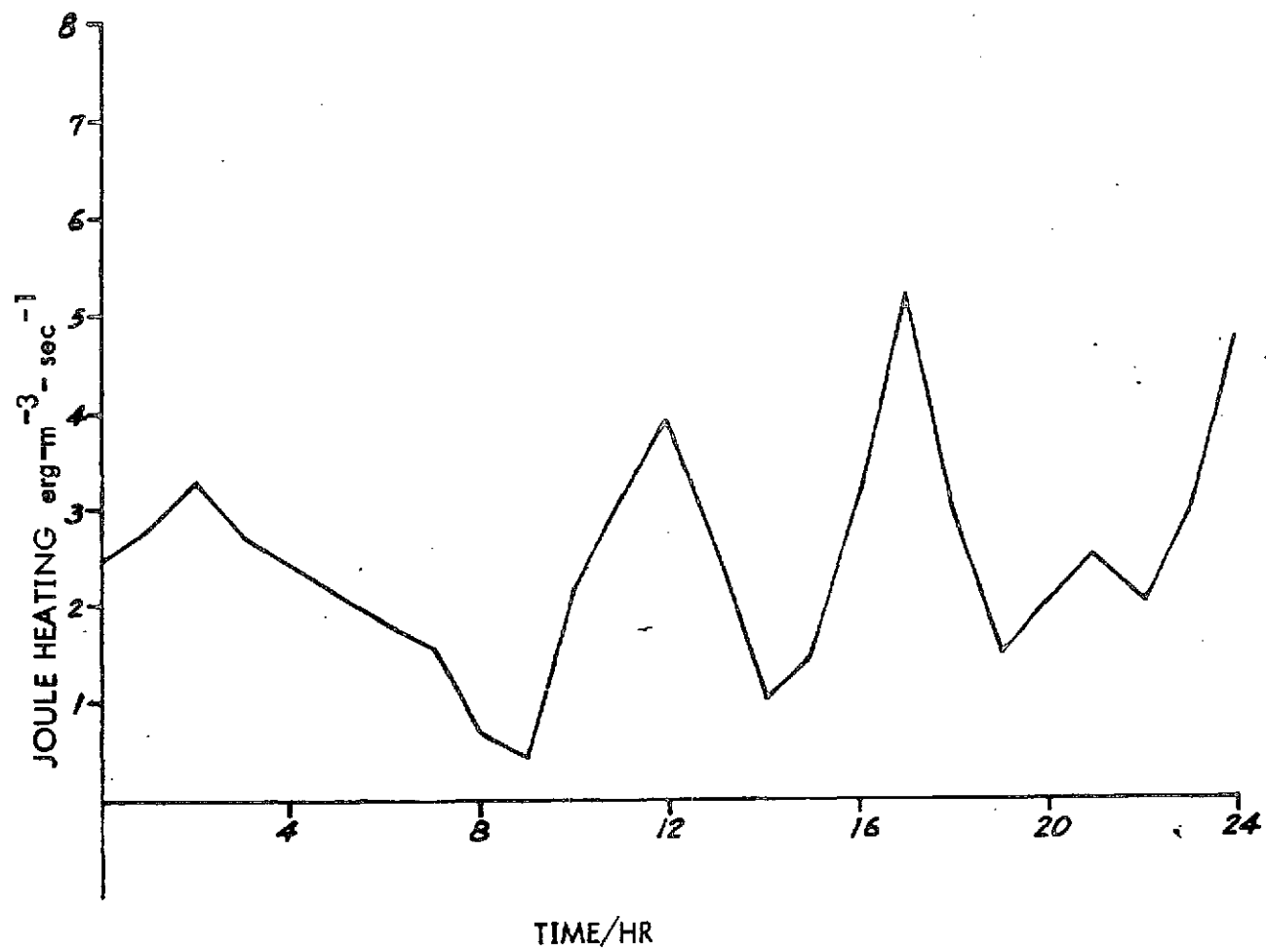


FIG. II-6

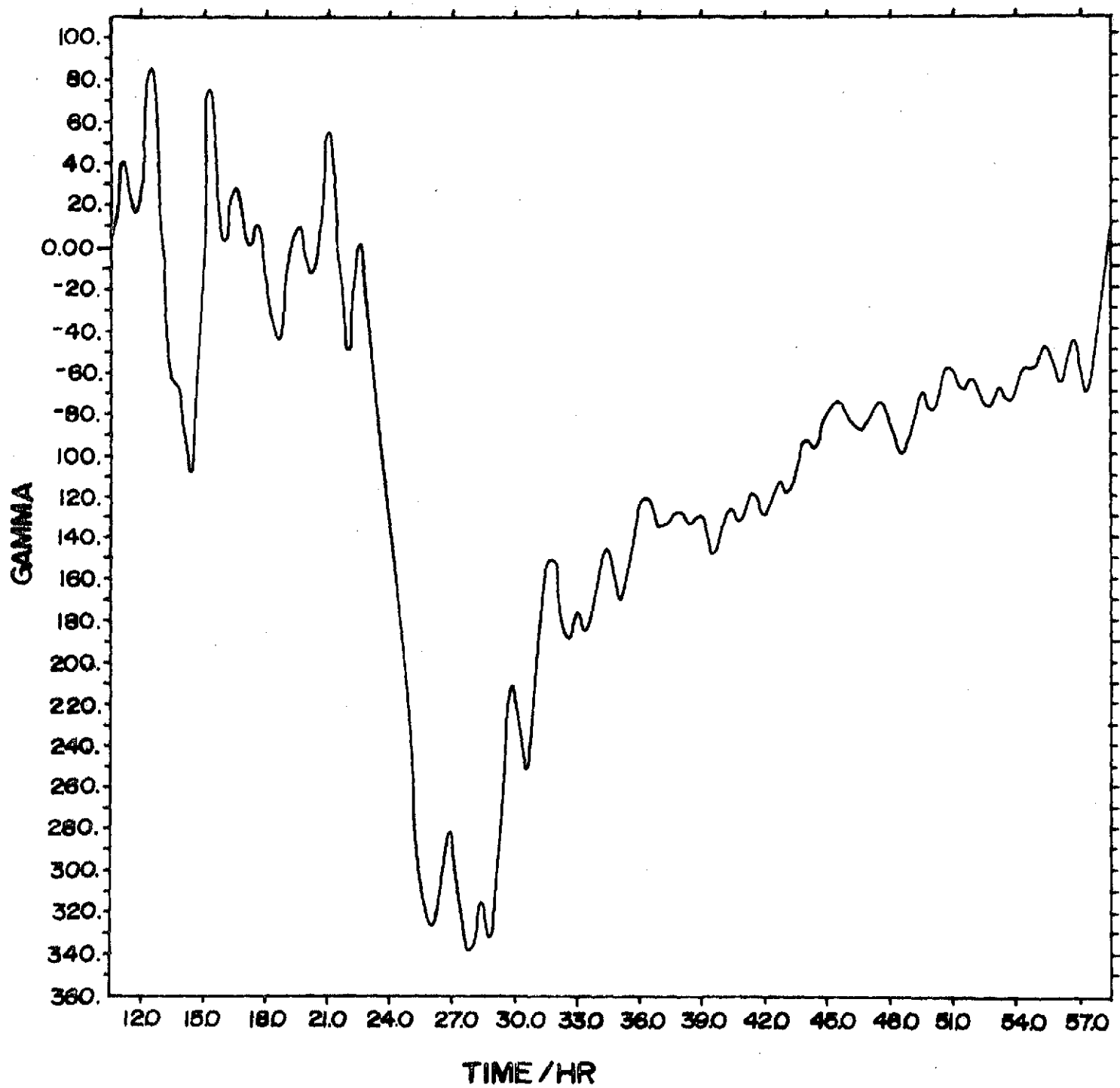


Fig II-7

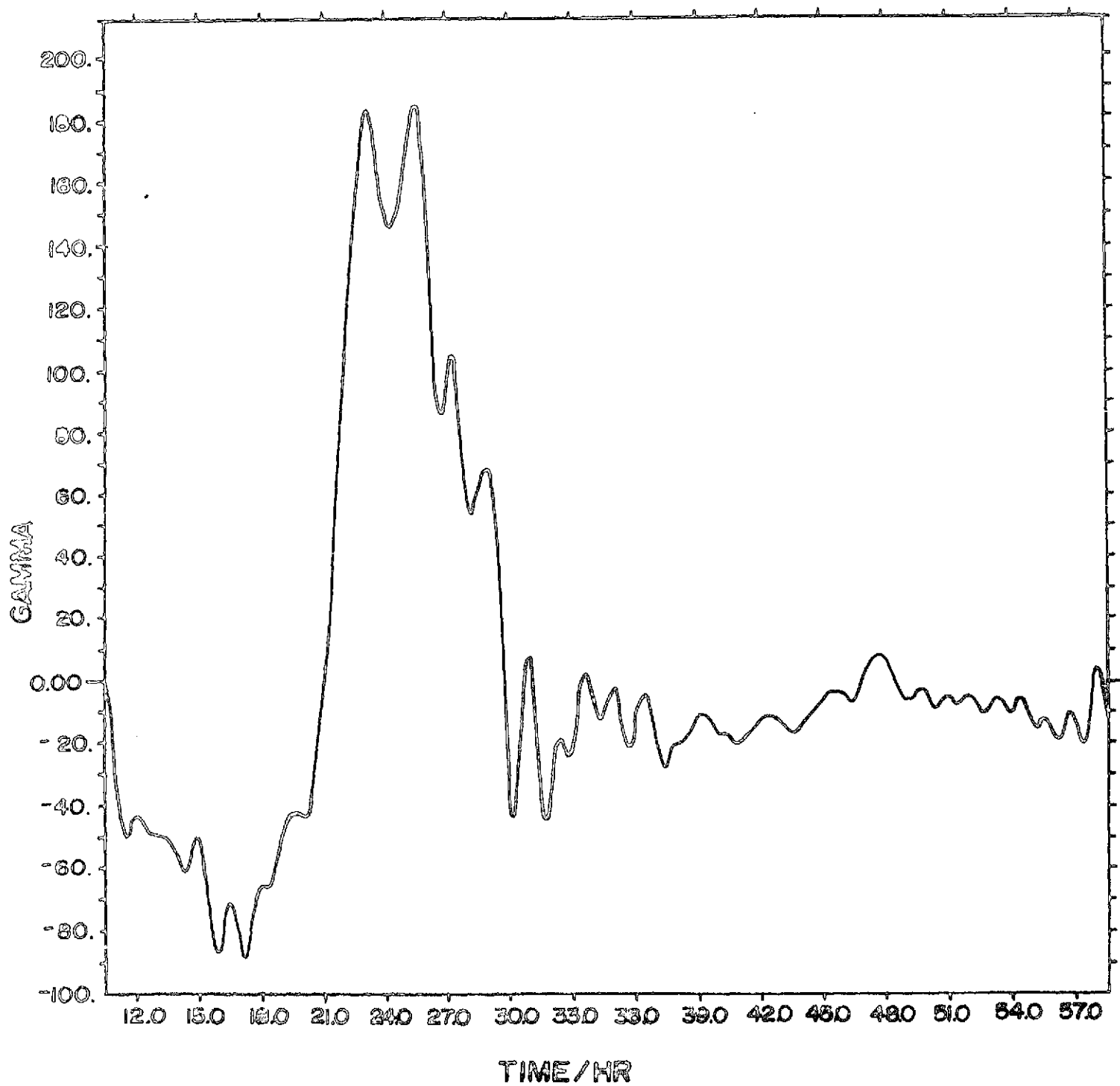


Fig. II-8

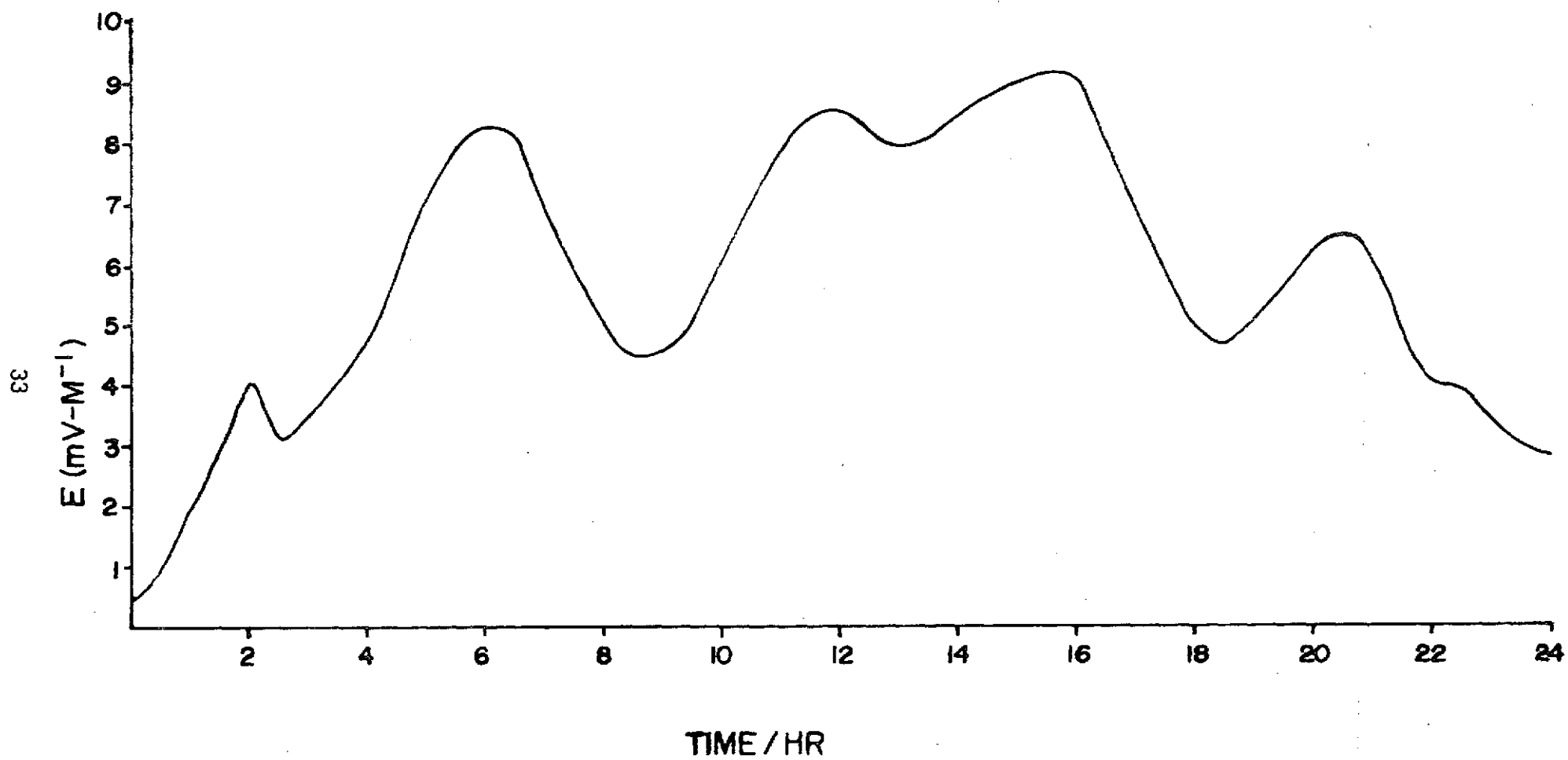


FIG. II-9

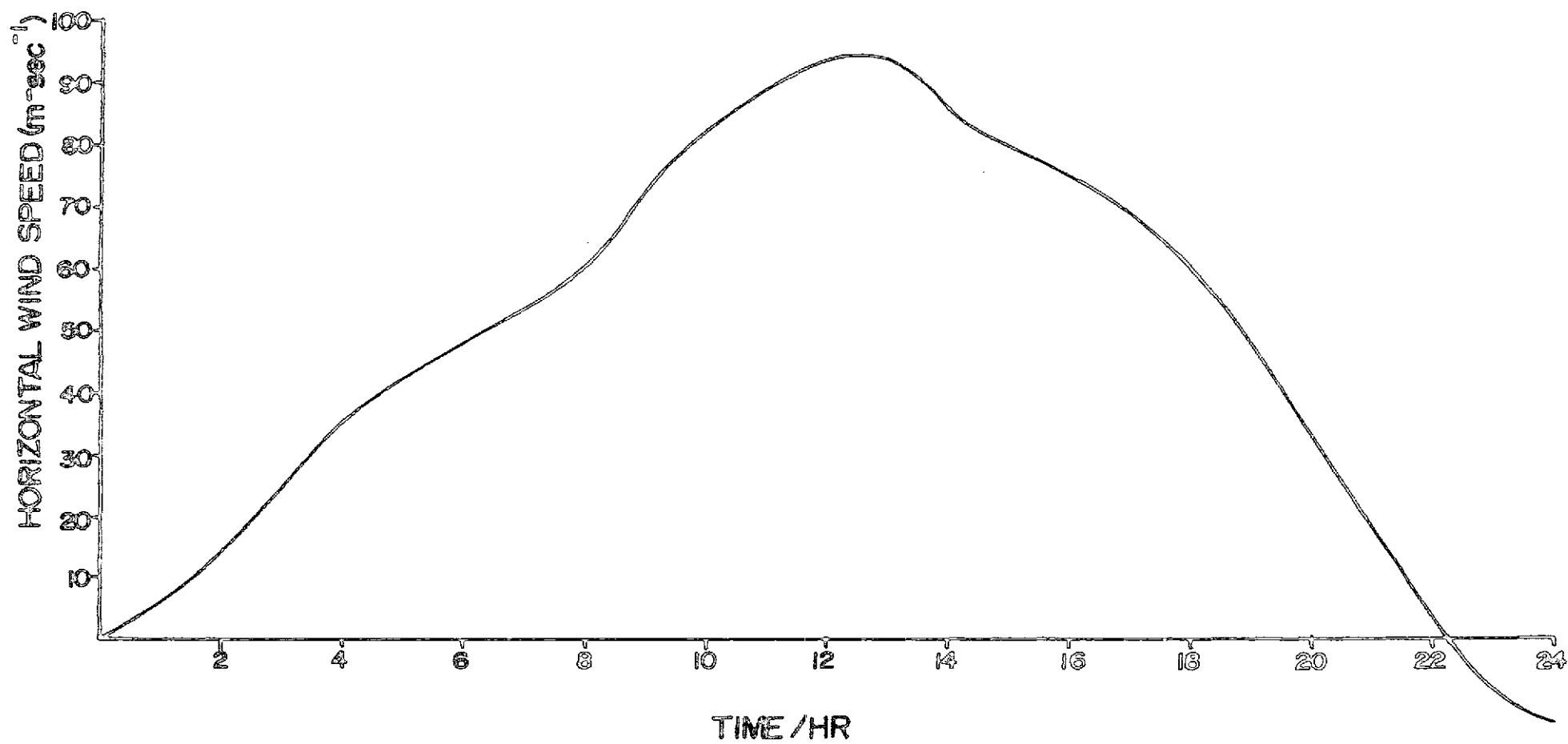


Fig II-10

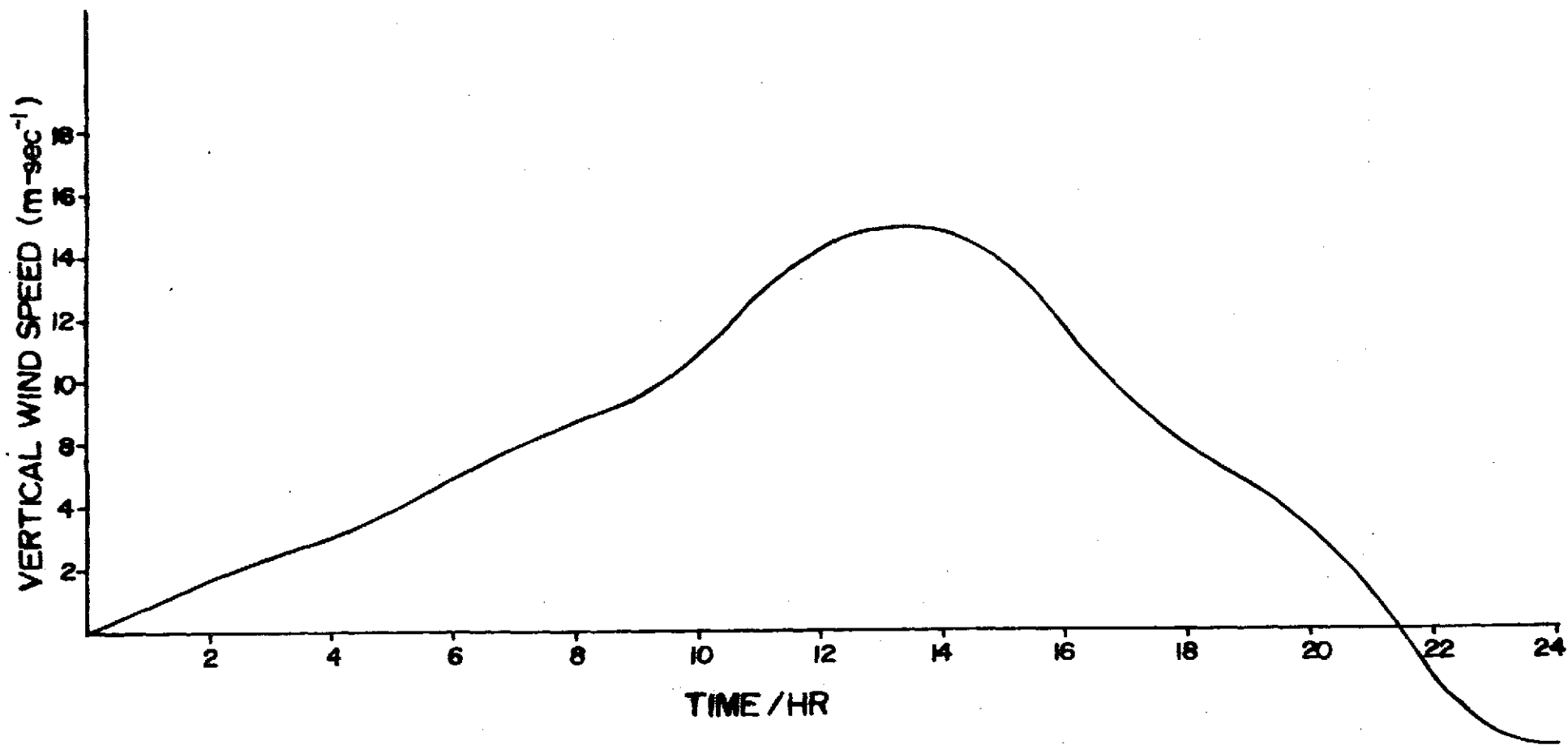


Fig. II-11

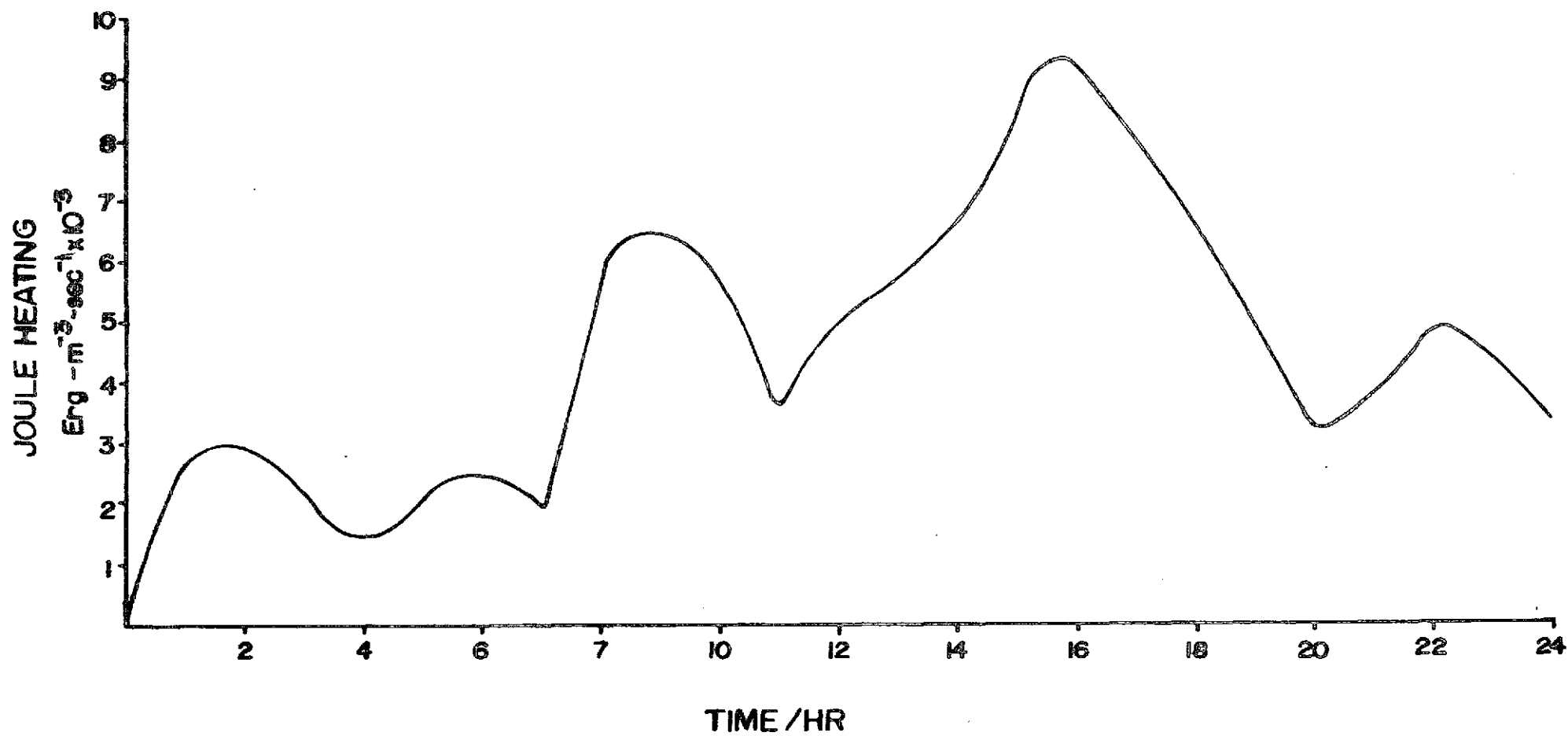


Fig. II-12



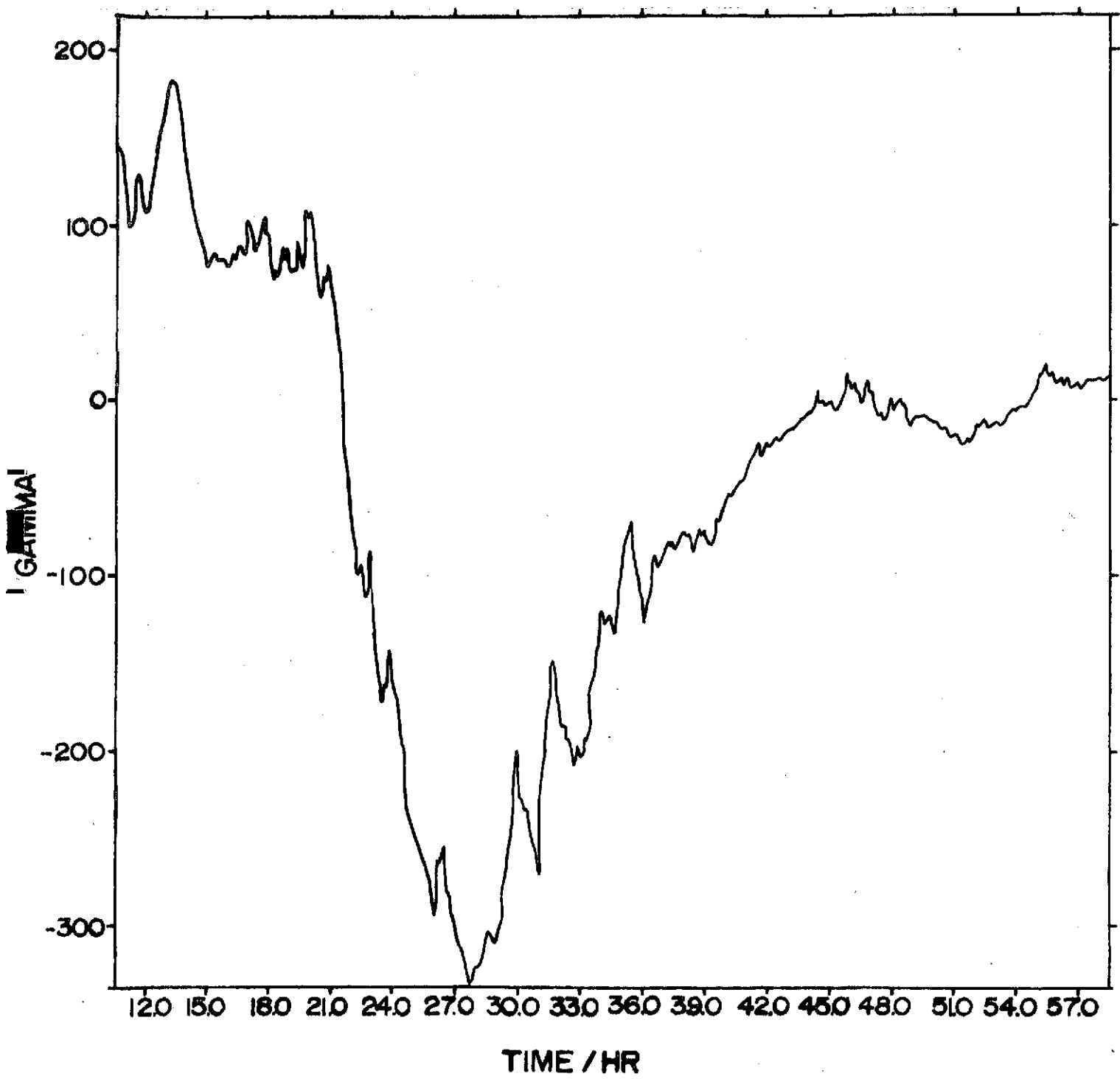


FIG. II-13

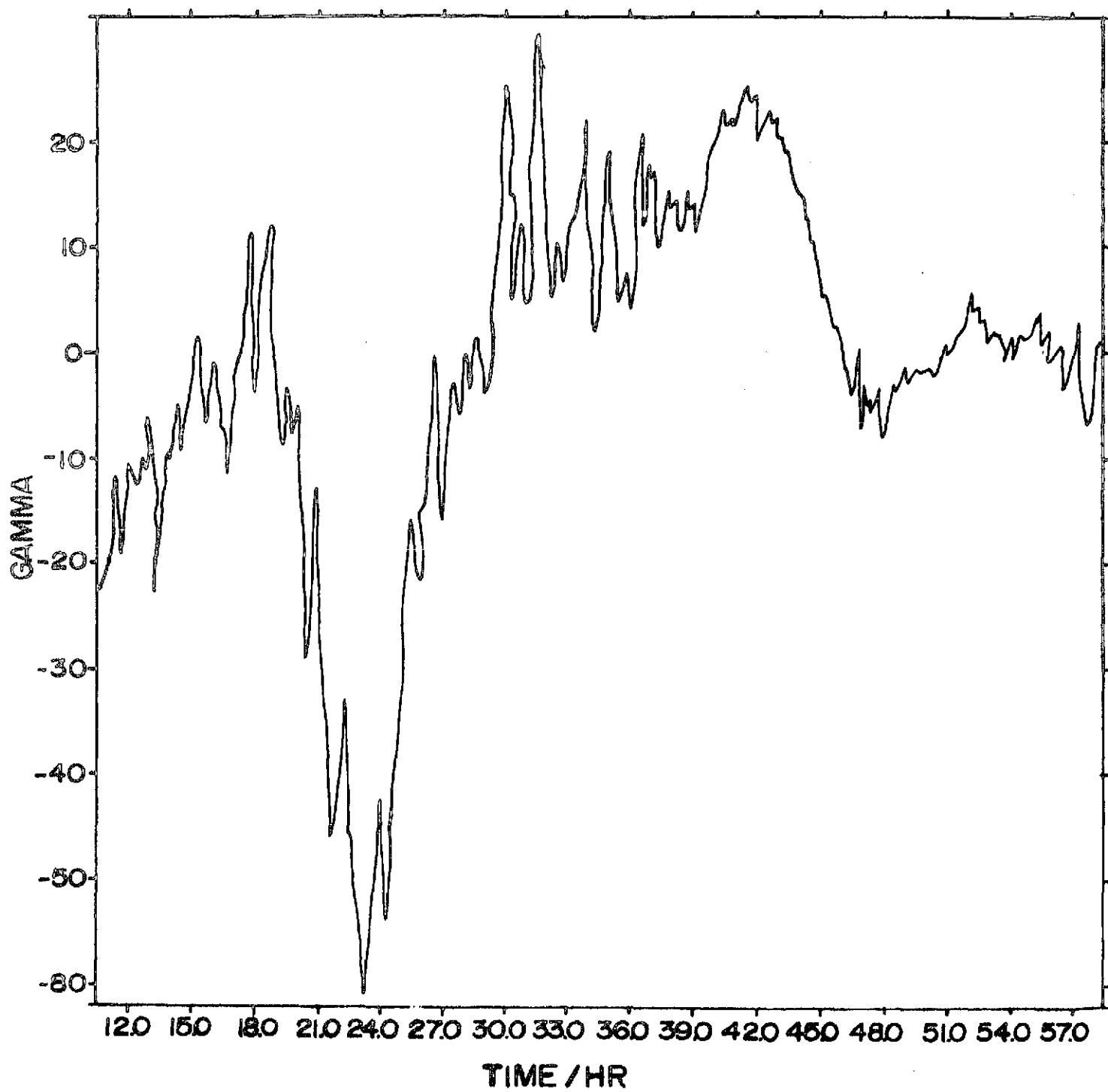


FIG. II-14

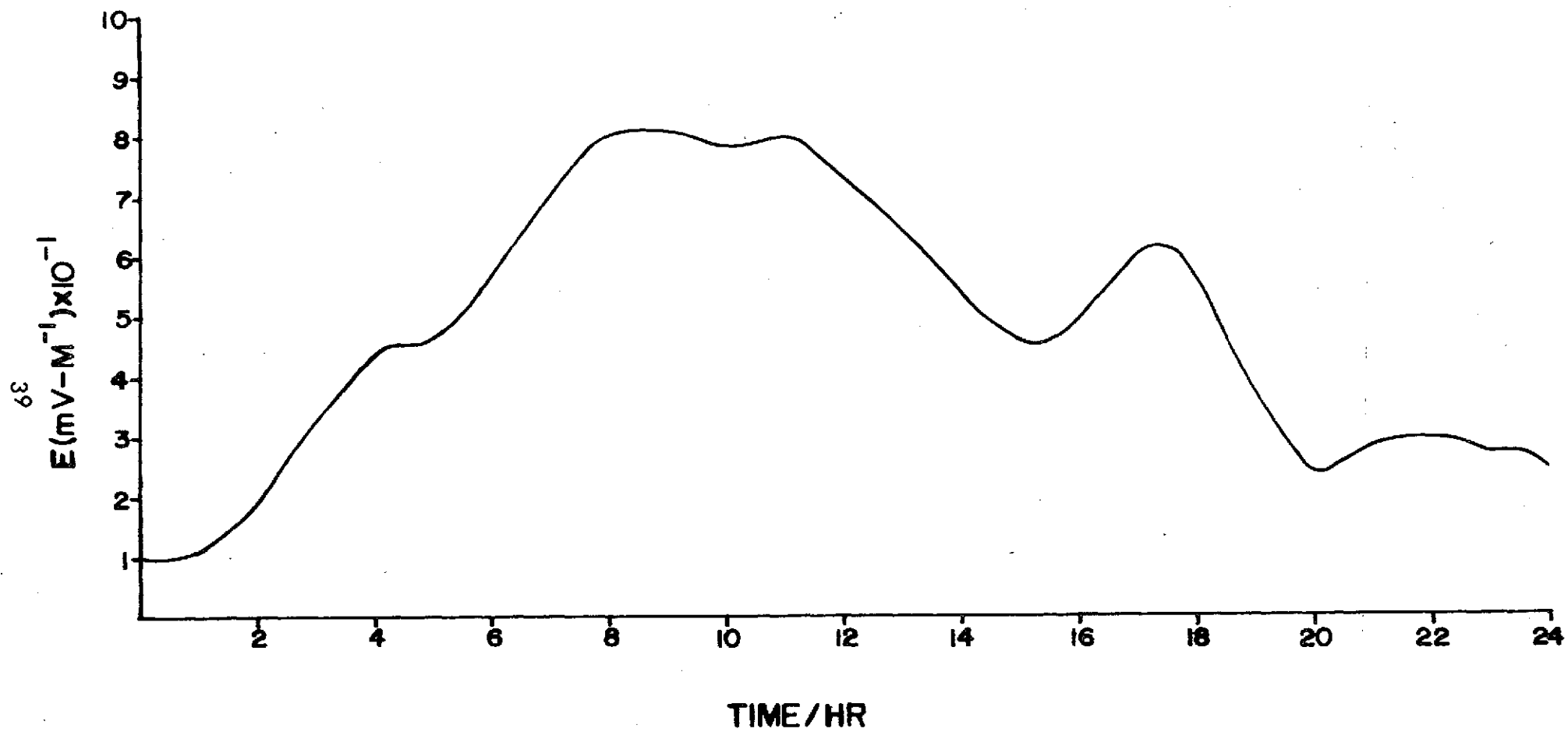


FIG. II-15

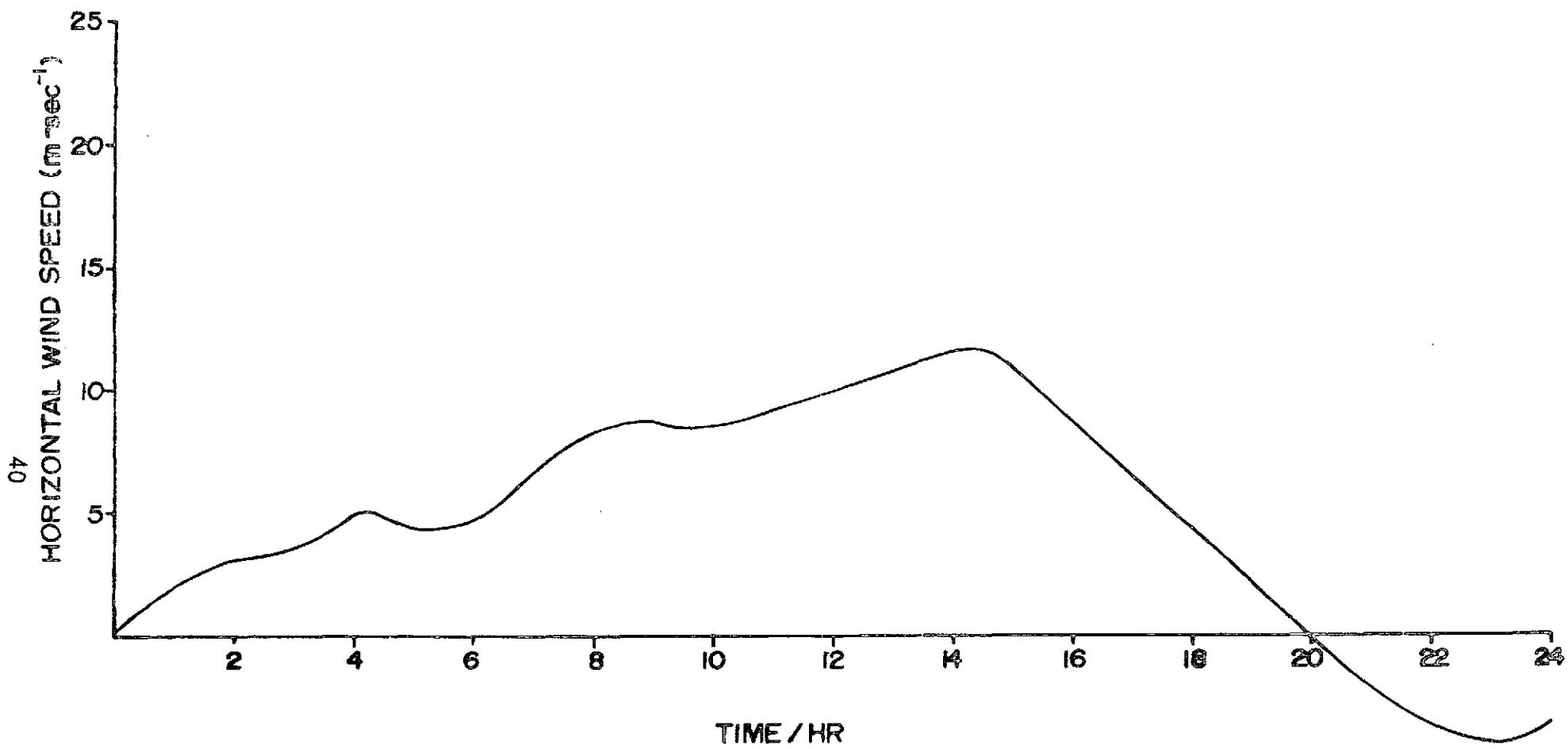
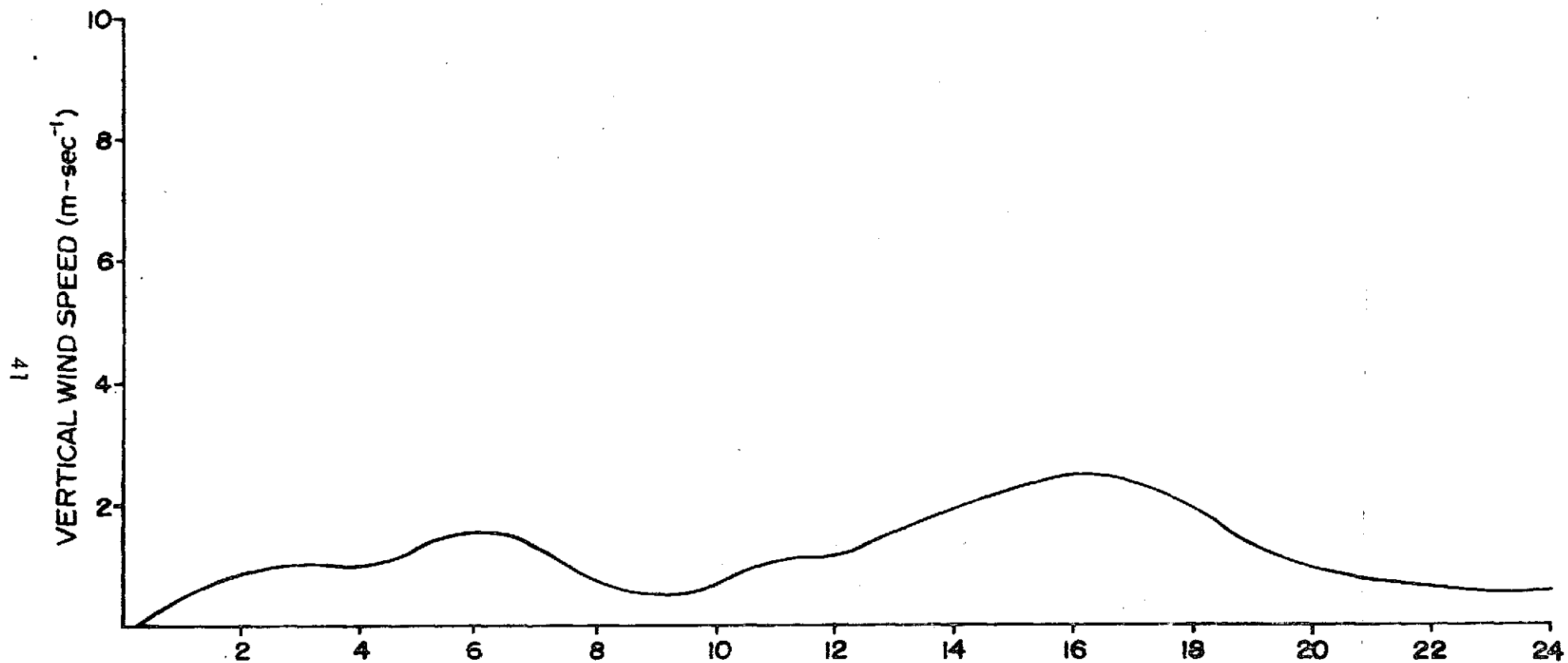


FIG. II-16



TIME / HR  
FIG. II-17

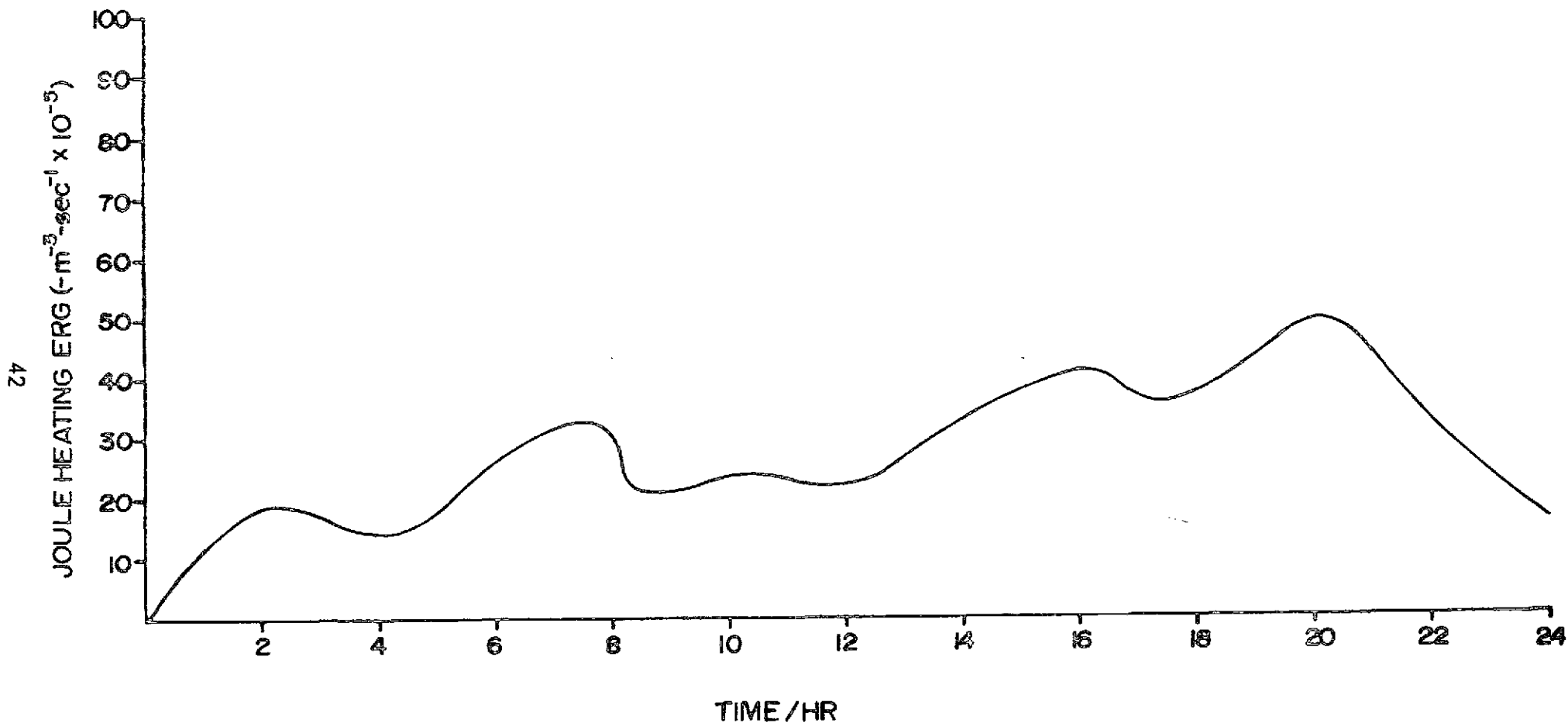


FIG. II-18

## CHAPTER III

### PROPAGATION OF HYDROMAGNETIC WAVES IN THE IONOSPHERE\*\*

#### III-1 Introduction

How hydromagnetic waves of solar origin heat and accelerate the solar wind has been one of the most exciting topics in solar physics in recent years (Hundhausen, 1970; Parker, 1971). In particular, the dissipation of magneto-acoustic waves of solar origin by the various transport phenomena in Coulomb collisions may be a major source of heating of the electrons in the outer solar corona, within five solar radii, which is a region of transition for electrons from a collisional to a collisionless regime (Hung and Barnes, 1972a; 1972b; 1972c). A similar procedure may be followed in studying the physics of the topside ionosphere.

In general, propagation of hydromagnetic waves is modified by transport phenomena due to Coulomb collisions. Collisional effects can vary from region to region in the plasma of interest. In particular, waves of a given period may see one region of the plasma as collisionless (in the sense that the wave period is short compared with the coulomb collision time) and another region of the plasma as collision-dominated. In studying the topside ionosphere, waves of period  $T \sim 10$  seconds are of special interest, because 10 seconds is the period of the typical ionospheric noise.\* Electron collision time at a height of 600 Km is 0.1 second, and at a height of 1000 Km is one second (Hanson, 1965). The ion collision time at 600 Km is 60 seconds (Hanson, 1965). Thus, for wave period of 10 seconds, in the altitude from 600 Km to 1000 Km, electrons are in a transitional regime between collisional and collisionless conditions while ions are in a collisionless regime.

---

\* Typical ionospheric noises such as hiss, whistler modes, micropulsations, etc., are those which have wave periods of the order of several seconds, while the ion gyrofrequency, under 3000 Km height, has a wave period of several milliseconds (Barrington and Fejer, 1965). In the present case, we chose the wave period of hydromagnetic wave as 10 seconds because it is believed that those ionospheric noises might be relevant to hydromagnetic waves.

\*\*This chapter was written with Dr. R. J. Hung.

It is therefore of considerable interest to investigate how hydromagnetic waves behave in this intermediate situation, and in particular, how they are dissipated and what can be the criteria of stability, if instability occurs. In this study we consider small amplitude waves in the situation in which electron-ion and electron-electron collisions (but not ion-ion collisions) must be taken into account. It is a reasonable approximation to assume that electrons follow fluid equations while the ions are adequately described by kinetic equations with ion-electron collisions.

In the lower altitude region of the ionosphere, sound waves play a significant role in modifying the atmospheric motions. Sound waves are important only when the wave frequency is much smaller than the collision frequency. As altitude increases, the total number density decreases and collision frequency decreases. When we reach the point where the collision frequency becomes smaller than the wave frequency, we are in the region where the ion-acoustic wave, which is the extreme case of the magneto-acoustic mode when the collision frequency tends to zero, becomes important. The dissipation mechanism affecting wave propagation also changes from transport effects due to Coulomb collisions for the collisional regime to Landau damping for the collisionless regime.

It is known that three different modes exist in hydromagnetic waves, namely, Alfvén, fast and slow modes (Landau and Lifshitz, 1960). In the laboratory, only fast and Alfvén modes have been observed (Schatzman, 1965). The nonexistence of slow mode in nature is because the damping rate of the slow mode is almost comparable to the wave frequency, so that in the laboratory there is no way to observe the slow mode before it is damped (Barnes, 1966). Thus, in this study, we are interested only in the less damped fast and Alfvén modes.

There is another long-period-wave with a period from 30 minutes to several hours called a gravity wave (Hines, 1960). The situation for gravity waves is quite different than for hydromagnetic waves. The propagation of a gravity wave in the lower ionosphere (mostly D and E region) is accompanied by the propagation of a pressure wave. Therefore, the condition for the existence of gravity waves is the same as the condition for the existence of pressure waves; namely, the collision frequency is greater than the wave frequency. Thus, in the present study, gravity waves are less important.



As shown in Sec. II, the electrons behave as a fluid whose heat balance is controlled by thermal conduction and viscous dissipation; and the ions behave much as if they were in a perfectly collisionless plasma, with a slight modification due to ion-electron collisions. In Sec. III and IV, we derive the dispersion relation which characterizes the behavior of hydromagnetic waves, and several special cases of wave propagation without using numerical calculation. The criteria for mirror and fire-hose instabilities are discussed for anisotropic ions in Sec. V. In Sec. VI, we discuss in detail the damping of magneto-acoustic waves under an assumed condition for the upper F2 region; the ions are heated by Landau damping in much the same way as in a perfectly collisionless plasma, and electrons are heated by conductive dissipation of waves at about 600 Km altitude and viscous dissipation of waves at about 1000 Km altitude. The implications of turbulent or irregular fluctuations of fields due to mirror and/or fire hose instabilities are also considered in Sec. VI.

The theory of the present paper breaks down when the wave period is shorter than the ion collision time or the wave period is longer than the electron collision time. The former condition will occur (for 10 second waves) at an altitude lower than 500 Km where both electrons and ions are governed by fluid equations. The latter condition may obtain in the ionosphere at altitudes higher than 1000 Km where the fluctuations in electron and ion components follow the kinetic equations for a collisionless plasma. Both of these limits will be analyzed in subsequent papers.

### III-2 Basic Equations: A Mixed Model

As mentioned above, in the upper F2 region, at altitudes of 600 Km and higher, electron-electron and electron-ion collisions can affect hydromagnetic waves with periods of 10 seconds. We idealize this region for the time scale of interest by treating the electrons as a fluid and assuming that the ions are adequately described by a kinetic theory that neglects ion-ion, but not ion-electron collisions. We further idealize the plasma and magnetic field by assuming that they are, on the average, uniform throughout an effectively infinite volume, that there is no net average electric current, and that the average electron pressure tensor is isotropic, but the ion pressure tensor is not. Let  $n$ ,  $\underline{v}$ ,  $T$ ,  $P$ ,  $\underline{q}$ , and  $\underline{\pi}$  denote number density, velocity, temperature, pressure, heat flux, and viscous stress tensor, respectively, and let subscripts and superscripts  $e$  and  $i$  denote electrons and ions, respectively. Let  $\underline{E}$  and  $\underline{B}$  be the electric and magnetic fields,  $e$  the ion charge,  $m_{(i)}$  the ion or electron mass, and  $c$  the speed of light. Then, the fluid equations for the electrons, and the kinetic equation for the ion velocity distribution  $f_i$  may be written as follows

(Braginskii, 1965):

$$\frac{\partial n_e}{\partial t} + \underline{\nabla} \cdot (n_e \underline{v}^e) = 0 \quad (2.1)$$

$$m_e n_e \frac{d \underline{v}^e}{dt} = - \underline{\nabla} P_e - \underline{\nabla} \cdot \underline{\pi}^e - e n_e (\underline{E} + \frac{1}{c} \underline{v}^e \times \underline{B}) + \underline{R}_e \quad (2.2)$$

$$\frac{3}{2} n_e \frac{dT_e}{dt} + P_e \underline{\nabla} \cdot \underline{v}^e = - \underline{\nabla} \cdot \underline{q}^e - \underline{\pi}^e : \underline{\nabla} \underline{v}^e \quad (2.3)$$

and

$$\begin{aligned}
\frac{\partial f_i}{\partial t} + \mathbf{v} \cdot \nabla f_i + \frac{e}{m_i} \left( \mathbf{E} + \frac{1}{c} \mathbf{v} \times \mathbf{B} \right) \cdot \frac{\partial f_i}{\partial \mathbf{v}} \\
= \frac{m_e}{m_i \tau_e} \frac{\partial}{\partial \mathbf{v}} \cdot \left( \mathbf{v} f_i + \frac{T_e}{m_i} \frac{\partial f_i}{\partial \mathbf{v}} \right) - \frac{1}{m_i n_i} \mathbf{R}_i \cdot \frac{\partial f_i}{\partial \mathbf{v}}
\end{aligned}
\quad (2.4)$$

where  $P_e = n_e T_e$ ,  $\left( \frac{\mathbf{P}}{\Pi} \right)^i = n_i \left( \frac{\mathbf{T}}{\Pi} \right)^i$ ,  $\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v}_e \cdot \nabla$ ,  $\tau_e$  is the electron collision time, and  $\mathbf{R}_e = -\mathbf{R}_i$  is the collisional momentum transfer from ions to electrons.  $\mathbf{R}$  is composed of a frictional force  $\mathbf{R}_U$  and a thermal force  $\mathbf{R}_T$ ,  $\mathbf{R} = \mathbf{R}_U + \mathbf{R}_T$ . The electron thermal flux  $\mathbf{q}_e$  is composed of analogous parts,  $\mathbf{q}_e = \mathbf{q}_e^U + \mathbf{q}_e^T$ . Finally, of course, the electromagnetic fields must satisfy Maxwell's equations.

In the present analysis, we are concerned with waves whose circular frequency  $\omega$  is small compared with the ion gyrofrequency, and whose wavelengths are long compared with the mean ion Larmor radius. We neglect the frictional force  $\mathbf{R}_U$  because  $\mathbf{R}_U$  is important only if  $\Omega_e \tau_e \ll 1$  (condition  $\Omega_e \tau_e \gg 1$  is always true for upper F-region ionosphere at 600 Km or higher), where  $\Omega_e = eB/m_e c$  is the electron gyrofrequency (cf. Hung and Barnes 1972a; 1972b). The momentum transfer due to collisions can be reduced to

$$\mathbf{R}_e = -0.71 n_e \nabla_{\parallel} T_e \quad (2.5)^*$$

where the subscripts  $\parallel, \perp$  refer to the magnetic field directions  $\mathbf{e}_z = \mathbf{B} / |\mathbf{B}|$ . Similarly to the present order of approximation, the electron heat flux is

$$\mathbf{q}_e = -K_e \nabla_{\parallel} T_e \quad (2.6)$$

where  $K_e$  is the coefficient of electron heat conductivity. Furthermore, the stress tensor (Braginskii 1965) is

---

\* Here the unit of  $T$  is the erg.

$$\underline{\pi}^e = - \underline{\eta}^e : \underline{w}^e \quad (2.7)$$

where the rate of strain tensor,  $\underline{w}^e$ , is

$$w_{\alpha\beta}^e = \frac{\partial v_{\alpha}^e}{\partial x_{\beta}} + \frac{\partial v_{\beta}^e}{\partial x_{\alpha}} - \frac{2}{3} \delta_{\alpha\beta} \frac{\partial v_{\alpha}^e}{\partial x_{\alpha}} \quad (2.8)$$

and  $\underline{\eta}^e$  is the tensor coefficient of electron viscosity, which is the function of  $\Omega_e$  and  $\tau_e$  and subscripts  $\alpha$  and  $\beta$  represent the coordinates. Under the present condition  $\Omega_e \tau_e \gg 1$ , the stress tensor  $\underline{\pi}^e$  has the following form in a coordinate system with  $z$ -axis parallel to the magnetic field (Hung and Barnes, 1972b; 1972c):

$$\pi_{zz}^e = -2 \eta_{vo}^e \left[ \frac{\partial v_z^e}{\partial z} - \frac{1}{3} \left( \frac{\partial v_x^e}{\partial x} + \frac{\partial v_y^e}{\partial y} + \frac{\partial v_z^e}{\partial z} \right) \right] \quad (2.9)$$

$$\pi_{xx}^e = \pi_{yy}^e = -\eta_{vo}^e \left[ \frac{\partial v_x^e}{\partial x} + \frac{\partial v_y^e}{\partial y} - \frac{2}{3} \left( \frac{\partial v_x^e}{\partial x} + \frac{\partial v_y^e}{\partial y} + \frac{\partial v_z^e}{\partial z} \right) \right] \quad (2.10)$$

$$\pi_{xy}^e \sim \pi_{yx}^e \sim \pi_{xz}^e \sim \pi_{zx}^e \sim \pi_{yz}^e \sim \pi_{zy}^e \sim \mathcal{O}(1/\Omega_e \tau_e)$$

where the zeroth-order coefficient of electron viscosity is

$$\eta_{vo}^e = 0.73 n_e T_e \tau_e \quad (2.11)$$

### III-2-1 Electron Dynamics

In order to solve the fluid-like equations, we assume

$$\begin{bmatrix} T \\ n \\ P \\ \underline{B} \\ \underline{E} \\ \underline{v}^e \end{bmatrix} = \begin{bmatrix} \langle T \rangle + \delta T(\underline{x}, t) \\ \langle n \rangle + \delta n(\underline{x}, t) \\ \langle P \rangle + \delta P(\underline{x}, t) \\ \langle \underline{B} \rangle + \delta \underline{B}(\underline{x}, t) \\ \delta \underline{E}(\underline{x}, t) \\ \delta \underline{v}^e(\underline{x}, t) \end{bmatrix} \quad (2.12)$$

where  $\langle \rangle$  denotes, ensemble averaging. We consider the limit of small amplitudes fluctuations,  $|\delta n| / \langle n \rangle \ll 1$ , etc. If the fluctuations are sinusoidal, i. e., proportional to  $\exp [i (\underline{k} \cdot \underline{x} - \omega t)]$  where  $i = \sqrt{-1}$ , Eqs. (2.1 - 2.3) become after linearization (and neglecting terms of order  $m_e / m_i$ )

$$\omega \delta n_e = \langle n_e \rangle (k_{\perp} \delta v_x^e + k_{\parallel} \delta v_z^e) \quad , \quad (2.13)$$

$$i \delta P_e k_{\perp} + \eta_{vo}^e \left( \frac{1}{3} k_{\perp}^2 \delta v_x^e - \frac{2}{3} k_{\perp} k_{\parallel} \delta v_z^e \right) + e \langle n_e \rangle \delta E_x + m_e \langle n_e \rangle \Omega_e \delta v_y^e = 0 \quad , \quad (2.14)$$

$$e \delta E_y - m_e \Omega_e \delta v_x^e = 0 \quad , \quad (2.15)$$

$$i \delta P_e k_{\parallel} + 2 \eta_{vo}^e \left( \frac{2}{3} k_{\parallel}^2 \delta v_z^e - \frac{1}{3} k_{\perp} k_{\parallel} \delta v_x^e \right) + e \langle n_e \rangle \delta E_z + 0.71 i \langle n_e \rangle k_{\parallel} \delta T_e = 0 \quad , \quad (2.16)$$

$$\langle P_e \rangle \underline{k} \cdot \delta \underline{v}_e = \left( \frac{3}{2} \omega \langle n_e \rangle + i k_{\parallel}^2 K_e \right) \delta T_e \quad . \quad (2.17)$$

Here, without loss of generality we have assumed  $\underline{k} = (k_{\perp}, 0, k_{\parallel})$ . Combining Eqs. (2.13) and (2.17), we have the following relation for pressure and number density

$$\delta P_e = \Gamma_e \langle T_e \rangle \delta n_e \quad (2.18)$$

where

$$\Gamma_e = 1 + \frac{2}{3 + 2 i \eta_{TC}} \quad (2.19)$$

$$\eta_{TC} = \frac{k_{\parallel}^2 K_e}{\omega \langle n_e \rangle} \quad (2.20)$$

To investigate the physical significance of Eq. (2.19), let us examine the parameter  $\Gamma$  in the following way:

$$\lim_{K_e \rightarrow 0} \Gamma = \frac{5}{3} \quad (2.21a)$$

$$\lim_{K_e \rightarrow \infty} \Gamma = 1 \quad (2.21b)$$

This means that in the two extreme cases of zero and infinite thermal conduction, the electrons fluctuate adiabatically and isothermally, respectively.

The velocity fluctuation of electrons can be easily found in terms of  $\delta \underline{E}$  by using Eqs. (2.13) and (2.18) in Eqs. (2.14 - 2.16):

$$\begin{bmatrix} \delta v_x^e \\ \delta v_y^e \\ \delta v_z^e \end{bmatrix} = \frac{c}{\Phi} \begin{bmatrix} 0 & M_{xy}^e & 0 \\ M_{yx}^e & M_{yy}^e & M_{yz}^e \\ 0 & M_{zy}^e & M_{zz}^e \end{bmatrix} \begin{bmatrix} \delta E_x \\ \delta E_y \\ \delta E_z \end{bmatrix} \quad (2.22)$$

where the components of the mobility tensor are

$$M_{xy}^e = -M_{yx}^e = 1,$$

$$M_{yy}^e = -\frac{k_{\perp}^2 \alpha_e^2 \zeta_v}{2 \omega \Omega_e} \left[ 1 + \frac{2(3\Gamma_e + i 2 \zeta_v)}{\alpha} \right],$$

$$M_{zz}^e = \frac{2 i \omega \Omega_e}{(1.71 \Gamma_e - 0.71) \alpha_e^2 k_{\parallel}^2} \left[ 1 + \frac{4 i \zeta_v}{\alpha} \right],$$

$$M_{zy}^e = -\frac{k_{\perp}}{k_{\parallel}} \left[ 1 + \frac{6 i \zeta_v}{\alpha} \right],$$

$$M_{yz}^e = \frac{k_{\perp}}{k_{\parallel}} \left( \frac{3\Gamma_e + 2i\zeta_v}{\alpha} \right),$$

where  $\alpha = 3(1.71\Gamma_e - 0.71) - 4i\zeta_v$ . Here

$$\zeta_v(\omega) = \frac{\omega \eta_{vo}^e}{\langle p_e \rangle} = 0.73 \omega \tau_e \quad (2.23)$$

is in general a complex function of  $\omega$ , and  $a_e = (2\langle T_e \rangle / m_e)^{1/2}$  is the electron thermal speed. From Eqs. (2.23), (2.20) and Braginskii's (1965) expression for  $K_e$ , it follows that

$$\eta_{TC} = 2.16 \frac{m_i}{m_e} \frac{\beta_e}{u^2} \zeta_v \quad (2.24)$$

where  $\beta_e = 8\pi \langle n \rangle \langle T_e \rangle / \langle B \rangle^2$ ,  $u = \omega / |k_{\parallel}| C_A$ , and  $C_A = \langle B \rangle / (4\pi m_i \langle n \rangle)^{1/2}$  Alfvén velocity. In deriving Eq. (2.22) we have exploited the fact that  $|\omega / \Omega_e| \ll 1$ .

### III-2-2 Ion Dynamics

Next, we consider the ion kinetic equation, with the object of finding an expression analogous to Eq. (2.22), for ions. The first term on the right hand side of Eq. (2.4) affects the evolution of the velocity distribution on the time scale  $\tau_e m_i / m_e \sim \tau_i (m_i / m_e)^{1/2}$  which is long compared with time scales of importance for the wave, and may therefore be neglected. As we pointed out earlier, the frictional force is also negligible. Hence, from Eq. (2.5), the right hand side of Eq. (2.4) is just

$$- \frac{1}{m_i n_i} R_i \cdot \frac{\partial f_i}{\partial \underline{v}} = - \frac{0.71}{m_i} (\underline{\nabla}_{\parallel} T_e) \cdot \frac{\partial f_i}{\partial \underline{v}} \quad (2.25)$$

Linearizing Equation (2.4), we obtain

$$(\underline{v} \times \langle \underline{B} \rangle) \cdot \frac{\partial \langle f_i \rangle}{\partial \underline{v}} = 0 \quad (2.26)$$

whose solution is

$$\langle f_i \rangle = f_0(v_{||}, v_{\perp}) \quad (2.27)$$

where  $f_0$  is arbitrary, and

$$\begin{aligned} & \left[ \frac{\partial}{\partial t} + (\underline{v} \cdot \nabla) + \frac{e}{m_i c} (\underline{v} \times \langle \underline{B} \rangle) \cdot \frac{\partial}{\partial \underline{v}} \right] \delta f_i \\ &= - \frac{e}{m_i} (\delta \underline{E} + \frac{1}{c} \underline{v} \times \delta \underline{B}) \cdot \frac{\partial \langle f_i \rangle}{\partial \underline{v}} - i \frac{0.71}{m_i} k_{||} \delta T_e \frac{\partial \langle f_i \rangle}{\partial v_{||}} \end{aligned} \quad (2.28)$$

Faraday's law

$$\frac{\omega}{c} \delta \underline{B} = \underline{k} \times \delta \underline{E} \quad (2.29)$$

Eqs. (2.13-22) permit us to write  $\delta \underline{B}$  and  $\delta T_e$  as linear combinations of the fluctuating electric field components. Then Eq. (2.28) can be solved by standard techniques to give an ion mobility tensor analogous to the electron mobility tensor of Eq. (2.22). However, it is simpler to exploit the fact that Eq. (2.28) is equivalent to the linearized Vlasov equation with  $\delta \underline{E}$  replaced by  $\delta \underline{E} + i (0.71/e) k_{||} \delta T_e \underline{e}_z$ . Hence, if  $\delta T_e$  is expressed in terms of  $\delta \underline{E}$ , we may easily find the ion mobility tensor from the Vlasov mobility tensor. In the present case, we have

$$\delta T_e = \frac{3i(\Gamma_e - 1)}{\alpha} \left[ \frac{e}{k_{||}} \delta E_z - \frac{k_{\perp}^2 a_e^2}{\omega \Omega_e} \zeta_v e \delta E_y \right], \quad (2.30)$$



Then, proceeding as in Hung and Barnes (1972a, b, c), we find

$$\delta \tilde{v}_i = \frac{e}{\langle B \rangle} M_i \cdot \delta \tilde{E} \quad (2.31)$$

where the mobility tensor with anisotropic ions are

$$M_{xx}^i = i \frac{\omega}{\Omega_i} \left[ \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} - 1 \right) \frac{1}{2 y_i^2} - 1 \right]$$

$$M_{yy}^i = i \frac{\omega}{\Omega_i} \left[ \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} - 1 \right) \frac{1}{2 y_i^2} - 1 + \frac{k_{\perp}^2}{k_{\parallel}^2} \frac{1}{2 y_i^2} \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \left( 2 + \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} Z_i' \right) \right]$$

$$M_{zz}^i = i \frac{\Omega_i}{\omega} y_i^2 Z_i' (1 - \Lambda)$$

$$M_{zy}^i = \frac{\beta_{\perp}^i}{2 \beta_{\parallel}^i} \frac{k_{\perp}}{k_{\parallel}} Z_i'$$

$$M_{yz}^i = - \frac{\beta_{\perp}^i}{2 \beta_{\parallel}^i} \frac{k_{\perp}}{k_{\parallel}} Z_i' (1 - \Lambda)$$

$$M_{xy}^i = - M_{yx}^i = 1 + \mathcal{O}\left(\frac{\omega^2}{\Omega_i^2}\right)$$

$$M_{xz}^i \sim M_{zx}^i \sim \mathcal{O}\left(\frac{\omega}{\Omega_i}\right)$$

where  $\Lambda = 2.13 (\Gamma_e - 1)/\alpha$ ,  $\Omega_i = e \langle B \rangle / m_i c$ ,  $\alpha_{\parallel}^i = (2 \langle T_{\parallel}^i \rangle / m_i)^{1/2}$ ,  $y_i = \omega / (|k_{\parallel}| \alpha_{\parallel}^i)$ , and  $Z_i = Z(y_i)$  and  $Z_i'$  are the plasma dispersion function and its first derivative (Fried and Conte 1961). In deriving Eq. (2.31), we have neglected the terms proportional to  $\Lambda \mathcal{O}_v Z_i'$  because

$$\wedge \zeta_v z_i' \leq \mathcal{O}\left(\frac{m_e}{m_i} \frac{\langle T_i \rangle}{\langle T_e \rangle}\right). \quad (2.32)$$

### III-3 Dispersion Relation

For the limit of hydromagnetic waves, the displacement current is always negligible. Then the electric current density may be written as

$$\underline{J} = - \frac{i \omega}{4 \pi} \underline{\underline{K}} \cdot \underline{E} \quad (3.1)$$

where the dielectric tensor

$$\underline{\underline{K}} = \frac{4 \pi i}{\omega} \frac{c}{\langle B \rangle} e \langle n \rangle (\underline{M}_i - \underline{M}_e) \quad (3.2)$$

From Eqs. (2.22), (2.31) and (3.2) we have

$$\underline{\underline{K}} = \begin{bmatrix} K_{xx} & 0 & 0 \left( \frac{\omega}{\Omega_i} \right) \\ 0 & K_{yy} & K_{yz} \\ 0 \left( \frac{\omega}{\Omega_i} \right) & K_{zy} & K_{zz} \end{bmatrix} \quad (3.3)$$

where the components of the dielectric tensor are

$$K_{xx} = - \left( \frac{c}{C_A} \right)^2 \left[ \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} - 1 \right) \frac{1}{2 y_i^2} - 1 \right]$$

$$K_{yy} = - \left( \frac{c}{C_A} \right)^2 \left[ \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} - 1 \right) \frac{1}{2 \gamma_i^2} - 1 + \frac{k_{\perp}^2}{k_{\parallel}^2} \frac{1}{2 \gamma_i^2} \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \left( 2 + \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} Z_i' \right) \right]$$

$$+ i \frac{k_{\perp}^2 \alpha_e^2 \omega_{pe}^2 \zeta_v}{2 \omega^2 \Omega_e^2} \left( 1 + \frac{2(3\Gamma_e + i 2 \zeta_v)}{\alpha} \right)$$

$$K_{zz} = - (1 - \Lambda) \frac{\omega_{pi}^2 Z_i'}{\alpha_{\parallel}^i k_{\parallel}^2} + \frac{2 \omega_{pe}^2}{\alpha_e^2 k_{\parallel}^2 \alpha}$$

$$K_{yz} = - i \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \right) \frac{k_{\perp} \omega_{pi}^2 Z_i' (1 - \Lambda)}{2 k_{\parallel} \omega \Omega_i} - i \frac{\omega_{pi}^2}{\omega \Omega_i} \frac{k_{\perp}}{k_{\parallel}} \frac{3\Gamma_e + i 2 \zeta_v}{\alpha}$$

$$K_{zy} = i \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \right) \frac{k_{\perp} \omega_{pi}^2 Z_i'}{2 k_{\parallel} \omega \Omega_i} + i \frac{\omega_{pi}^2}{\omega \Omega_i} \frac{k_{\perp}}{k_{\parallel}} \left( 1 + \frac{i 6 \zeta_v}{\alpha} \right)$$

$$\omega_{p(i)}^2 = 4 \pi \langle n \rangle e^2 / m_{(i)}.$$

The requirement that Eq. (3.1) be consistent with Maxwell's equations gives the dispersion relation

$$\det \left[ \left( \frac{c}{\omega} \right)^2 (k_{\perp} k_{\perp} - k_{\parallel}^2 \mathbf{1}) + K \right] = 0 \quad (3.4)$$

Substituting Eq. (3.3) in Eq. (3.4) gives

$$\left[ K_{xx} - \left( \frac{c k_{\parallel}}{\omega} \right)^2 \right] \left[ \left( K_{yy} - \frac{k_{\perp}^2 c^2}{\omega^2} \right) \left( K_{zz} - \frac{k_{\perp}^2 c^2}{\omega^2} \right) - K_{yz} K_{zy} \right] = 0 \quad (3.5)$$

to lowest order in  $\omega/\Omega_i$ . The first factor is the usual Alfvén wave dispersion relation

$$\left(\frac{\omega}{k_{||} c_A}\right)^2 = 1 + \frac{1}{2} (\beta_{\perp}^i - \beta_{||}^i) \quad (3.6)$$

and the second factor, which gives the magneto-acoustic dispersion relation, is

$$\left(K_{yy} - \frac{k^2 c^2}{\omega^2}\right) K_{zz} - K_{zy} K_{yz} = 0 \quad (3.7)$$

since  $|K_{zz}| \gg |k c / \omega|^2$ .

By using Eq. (3.3), we can rearrange Eq. (3.7) by straightforward calculation to give

$$\left[1 + \frac{1}{2} (\beta_{\perp}^i - \beta_{||}^i) - u^2\right] \cot^2 \theta = S(u, \omega \tau_e) \quad (3.8)$$

where

$$\begin{aligned} S(u, \omega \tau_e) = & 1 + \beta_{\perp}^i + \frac{1}{2} \frac{(\beta_{\perp}^i)^2}{\beta_{||}^i} Z_i' - i \frac{\beta_e \zeta_v}{2} \left(1 + \frac{2(3\Gamma_e + i2\zeta_v)}{\alpha}\right) \\ & - \frac{\beta_e \beta_{||}^i}{4\alpha} \left[ \left(\frac{\beta_{\perp}^i}{\beta_{||}^i}\right) (\Gamma_e - i\frac{4}{3}\zeta_v) Z_i' + 2(\Gamma_e + i\frac{2}{3}\zeta_v) \right] \left[ \alpha \left(\frac{\beta_{\perp}^i}{\beta_{||}^i}\right) Z_i' + 2(\alpha + i6\zeta_v) \right] \\ & \quad \beta_e (\Gamma_e - i\frac{4}{3}\zeta_v) Z_i' - 2\beta_{||}^i \end{aligned} \quad (3.9)$$

$$\alpha = 3(1.71\Gamma_e - 0.71) - 4i\zeta_v \quad (3.10)$$

$$\zeta_v = 0.73 \omega \tau_e \quad (3.11)$$

$$\Gamma_e = 1 + 2 / (3 + 2i \eta_{TC}), \eta_{TC} = 2.16 \left( \frac{m_i}{m_e} \right) \frac{\beta_e}{u^2} \zeta_v \quad (3.12)$$

$$\beta_e = \frac{8\pi}{\langle B \rangle^2} \langle p_e \rangle, \quad \beta_{\left( \frac{L}{\Pi} \right)}^i = \frac{8\pi}{\langle B \rangle^2} \langle p_{\left( \frac{L}{\Pi} \right)}^i \rangle \quad (3.13)$$

$$\theta = \chi(k, \langle \underline{B} \rangle) \quad (3.14)$$

$$\text{and} \quad u = \omega / (|k_{\parallel}| C_A) \quad (3.15)$$

We notice that if dispersion relation (3.8) is satisfied for a wave  $(\omega, \underline{k})$ , it is also satisfied for a wave propagating in the opposite direction, i.e., for the waves  $(\omega, -\underline{k})$  and  $(-\omega^*, \underline{k})$ . This is so first, because (3.8) is invariant to the transformation  $(\omega, \underline{k}) \rightarrow (\omega, -\underline{k})$  and secondly, because the transformation  $(\omega, \underline{k}) \rightarrow (-\omega^*, \underline{k})$  causes  $\eta_{TC} \rightarrow -\eta_{TC}^*$ ,  $\zeta_v \rightarrow -\zeta_v^*$ ,  $\Gamma_e \rightarrow \Gamma_e^*$ ,  $Z_i' \rightarrow Z_i'^*$ ,  $u^2 \rightarrow (u^*)^2$ , so that Eq. (3.8) is transformed into complex conjugate. Hence, in particular, we may always choose  $\text{Re } \eta_{TC} \geq 0$  and  $\text{Re } \zeta_v \geq 0$ .

Note the superscript \* indicates the complex conjugate.

### III-4 Special Cases of the Dispersion Relation

Eq. (3.6) gives the dispersion relation for Alfvén waves in a plasma with anisotropic ions. The analog of Eq. (3.6) in a purely collisionless plasma is

$$\left( \frac{\omega}{k_{\parallel} C_A} \right)^2 = 1 + \frac{1}{2} (\beta_{\perp} - \beta_{\parallel}) \quad (4.1)$$

where  $\beta_{\parallel} = \beta_{\parallel}^e + \beta_{\parallel}^i$ . For isotropic electrons,  $\langle T_{\parallel}^e \rangle \sim \langle T_{\perp}^e \rangle$  or  $\beta_{\parallel}^e \sim \beta_{\perp}^e$ , Eq. (4.1) is reduced to Eq. (3.6). If the ion collision frequency is high enough to make the ion pressure tensor isotropic, the dispersion relation (3.6) becomes identical to the transverse Alfvén wave of magnetohydrodynamics.

The magneto-acoustic dispersion relation (3.8), which is much more complicated than either the collisionless or collision-dominated cases, is transcendental, and must generally be solved numerically. However, we first consider a few interesting special limits that can provide insight into the character of the waves without numerical calculations. In the present case,  $S$  depends, in general, on  $\omega$  and  $\theta$  through  $\eta_{TC}$  and  $\zeta_v$  which are functions of the electron-electron collision time  $\tau_e$ , and  $\tau_e$  depends on the altitude of the ionosphere.

#### III-4-1 Propagation Parallel to $\langle \underline{B} \rangle$

When  $\underline{k} \parallel \langle \underline{B} \rangle$ ,  $\cot^2 \theta \rightarrow 0$ , so that either left-hand side of Eq. (3.8) becomes zero, i.e.,

$$\left( \frac{\omega}{k_{\parallel} C_A} \right)^2 = 1 + \frac{1}{2} (\beta_{\perp}^i - \beta_{\parallel}^i) \quad (4.2)$$

or  $S \rightarrow \infty$  ( $v^2$  finite), i.e.,

$$Z_i' = \frac{2 \langle T_{\parallel}^i \rangle}{\langle T_e \rangle} \frac{1}{\Gamma_e - i \frac{4}{3} \zeta_v} \quad (4.3)$$

Eq. (4.2) shows that one of the magneto-acoustic modes propagates as an Alfvén wave when  $\theta = 0$  in a plasma with anisotropic ions. Eq. (4.3) corresponds to a compressive wave propagating along  $\langle \underline{B} \rangle$ . The analogous wave in collisional magnetohydrodynamics is the sound wave, and in a collisionless plasma the analogous wave is the ion-acoustic wave, which generally undergoes Landau damping (Fried and Gould, 1961). When  $\omega \tau_e$  is on the order of or smaller than unity, which corresponds to the altitude of about 1000 Km in the ionosphere,  $\zeta_v$  is also on the order of or smaller than unity, so it leads to

$$|\eta_{TC}| \gg 1 \text{ and } \Gamma_e \approx 1. \quad (4.4)$$

This means that when the electron collision frequency is on the order of the wave frequency, or at an altitude of about 1000 Km, the viscous dissipation is more effective than conductive dissipation. Under this condition, Eq. (4.3) becomes

$$Z'_i \approx \frac{2 \langle T_{||}^i \rangle}{\langle T_e \rangle} \frac{1 + i \frac{4}{3} \zeta_v}{1 + \left| \frac{4}{3} \zeta_v \right|^2} \quad (4.5)$$

If Eq. (4.5) has any weakly damped solution, i.e., solutions such that  $|\text{Im } \omega| / \text{Re } \omega \ll 1$ , and  $S_v \approx S_v^0 = \text{Re } S_v$ , we would have

$$\text{Re } Z'_i \approx \frac{2 \langle T_{||}^i \rangle}{\langle T_e \rangle} \frac{1}{1 + \left( \frac{4}{3} \zeta_v^0 \right)^2} \quad (4.6a)$$

$$\text{Im } Z'_i \approx \frac{2 \langle T_{||}^i \rangle}{\langle T_e \rangle} \frac{\frac{4}{3} \zeta_v^0}{1 + \left( \frac{4}{3} \zeta_v^0 \right)^2} \quad (4.6b)$$

Since  $\text{Im } Z'_i \leq 0$  for real arguments, and Eq. (4.6b) cannot be satisfied for  $|\text{Im } \omega| / \omega \ll 1$  unless

$$\text{Re } u \gg \beta_{||}^i \quad (4.7)$$



Therefore, Eqs. (4.6a) and (4.6b) become

$$\left(\frac{\omega}{k}\right)^2 \approx \frac{\langle T_e \rangle}{m_i} \quad (4.8a)$$

$$\frac{\text{Im } \omega}{\text{Re } \omega} \approx -0.49 \omega \tau_e \quad (4.8b)$$

which are valid only if

$$\beta_e \gg \beta_i \quad (4.9)$$

This condition is nothing but  $\langle T_e \rangle \gg \langle T_i \rangle$  which is just the condition for weak damping of ion acoustic wave in a perfectly collisionless plasma (Fried and Gould, 1961).

Next, let us consider the other limit, i.e., when  $\omega \tau_e$  is on the order of or smaller than  $(m_e / m_i)$ , which corresponds to an altitude of about 600 Km or higher in the ionosphere. Under this limit, conductive dissipation becomes significant. Therefore, Eq. (4.3) becomes

$$Z_i' = \frac{2 \langle T_{||}^i \rangle}{\langle T_e \rangle} \frac{15 + 4 |\eta_{TC}|^2 + 4 i \eta_{TC}}{25 + 4 |\eta_{TC}|^2} \quad (4.10)$$

where  $\eta_{TC} = k_{||}^2 K_e / \omega \langle n_e \rangle$ . For the weakly damped solutions, i.e.,

$|\text{Im } \omega / \text{Re } \omega| \ll 1$ , and  $\eta_{TC} \approx \eta_{TC}^0 = \text{Re } \eta_{TC}$ , we would have

$$\text{Re } Z_i' \approx \frac{2 \langle T_{||}^i \rangle}{\langle T_e \rangle} \frac{15 + 4 |\eta_{TC}^0|^2}{25 + 4 |\eta_{TC}^0|^2} \quad (4.11a)$$

$$\text{Im } Z_i' \approx \frac{2 \langle T_{||}^i \rangle}{\langle T_e \rangle} \frac{4 \eta_{TC}^0}{25 + 4 |\eta_{TC}^0|^2} \quad (4.11b)$$

Using the same argument as in Eq. (4.7), Eqs. (4.11a) and (4.11b) become

$$\left(\frac{\omega}{k_{||}}\right)^2 \simeq \frac{25 + 4 \eta_{TC}^o}{15 + 4 \eta_{TC}^o} \frac{\langle T_e \rangle}{m_i} \quad (4.12a)$$

$$\frac{\text{Im } \omega}{\text{Re } \omega} \simeq - \frac{2 \eta_{TC}^o}{15 + 4 \eta_{TC}^o} \quad (4.12b)$$

which are valid only if

$$\frac{2 \eta_{TC}^o}{15 + 4 \eta_{TC}^o} \ll 1 \quad (4.13)$$

Particularly, in the adiabatic limit  $\eta_{TC}^o \rightarrow 0$ , corresponding to E-region or lower in the ionosphere,

$$\left(\frac{\omega}{k}\right)^2 \simeq \frac{5}{3} \frac{\langle T_e \rangle}{m_i} \quad , \quad (4.14a)$$

$$\frac{\text{Im } \omega}{\text{Re } \omega} \simeq - \frac{2}{15} \eta_{TC}^o \quad , \quad (4.14b)$$

are sonic waves propagating parallel to  $\langle \tilde{B} \rangle$ . In the isothermal limit  $\eta_{TC}^o \rightarrow \infty$ , it corresponds to 2000 Km altitude in the ionosphere,

$$\left(\frac{\omega}{k}\right)^2 \simeq \frac{\langle T_e \rangle}{m_i} \quad , \quad (4.15a)$$

$$\frac{\text{Im } \omega}{\text{Re } \omega} \simeq - \frac{1}{2} \eta_{TC}^o \quad , \quad (4.15b)$$

which results in an equation similar to Eq. (4.8) although the damping rates are different.

In summary, we have shown that viscous damping is significant at the higher altitude (about 1000 Km), and conductive dissipation is overwhelming in lower altitudes (600 Km or higher) of the ionosphere. In practice, all ion-acoustic waves propagating parallel to  $\langle \underline{B} \rangle$  above the F region, and sonic waves at E-region or lower, are dissipated, only if that the case of  $\langle T_e \rangle \gg \langle T_i \rangle$  will assure the propagation of the weakly damped ion-acoustic waves.

### III-4-2 Propagation Transverse to $\langle \underline{B} \rangle$

When  $\underline{k} \perp \langle \underline{B} \rangle$ ,  $Z_i \rightarrow 0$  and  $\eta_{TC} \rightarrow 0$ . Under this condition,  $\Gamma_e$  becomes 5/3 and expression (3.8) simplifies considerably.

$$\left( \frac{\omega}{k C_A} \right)^2 \simeq 1 + \beta_{\perp}^i - \beta_e \left( \frac{5}{6} - i \frac{\zeta_v}{6} \right) \quad (4.16)$$

Therefore, we have

$$\left( \frac{\omega}{k} \right)^2 \simeq C_A^2 + \frac{2 \langle T_{\perp}^i \rangle}{m_i} + \frac{5}{3} \frac{\langle T_e \rangle}{m_i} - i \frac{\zeta_v}{3} \frac{\langle T_e \rangle}{m_i} \quad (4.17)$$

If  $\zeta_v$  is small, and  $\zeta_v^0 = \text{Re } \zeta_v \gg \text{Im } \zeta_v$ , then the real part of (4.17) is

$$\left( \frac{\omega}{k} \right)^2 \simeq C_A^2 + \frac{2 \langle T_{\perp}^i \rangle + \frac{5}{3} \langle T_e \rangle}{m_i} \quad (4.18)$$

and

$$\frac{\text{Im } \omega}{\text{Re } \omega} \simeq - \frac{\zeta_v^0}{6} \frac{k^2}{(\text{Re } \omega)^2} \frac{\langle T_e \rangle}{m_i} \quad (4.19)$$

Eq. (4.18) is intermediate between the analogous expression which arises when ions are collisional

$$\left( \frac{\omega}{k} \right)^2 = C_A^2 + \frac{5}{3} \frac{\langle T_i \rangle + \langle T_e \rangle}{m_i} \quad (4.20)$$

and that which arises when electrons are collisionless

$$\left(\frac{\omega}{k}\right)^2 = C_A^2 + 2 \frac{\langle T_{\perp}^i \rangle + \langle T_{\perp}^e \rangle}{m_i} \quad (4.21)$$

In a collisionless plasma, strong electron Landau damping can occur for  $\theta$  slightly different from  $\pi/2$  (Barnes 1966). However, when  $\underline{k}$  is exactly perpendicular to  $\langle \underline{B} \rangle$ , this Landau damping vanishes. At the lower altitude of the F-region where conductive damping overwhelms viscous damping, conductive dissipation vanishes at  $\theta = \pi/2$ . In the case of a higher altitude, say 1000 Km, where the viscous damping becomes significant, the result is different. No matter how small the coefficient of electron viscosity is, as long as  $\zeta_v^0$  is not zero, viscous dissipation always can be found even when  $\underline{k}$  is exactly perpendicular to  $\langle \underline{B} \rangle$ .

### III-5 Criteria for Mirror and Fire-Hose Instabilities

Most of the known plasma instabilities, which are termed overstabilities, are a kind in which  $\omega$  is complex rather than pure imaginary (Stix, 1962). The non-occurrence of overstable solutions has been shown by Vedenov and Sagdeev (1958), Chandrasekhar et al (1958), and Barnes (1966) for the case of purely collisionless plasma. The significance of the nonexistence of overstable solutions is simply that the condition of marginal stability for the plasma is given by the dispersion relation with  $\omega = 0$ . This conclusion remains valid for the case of collisional electrons and collisionless ions.

Let us rewrite dispersion relation (3.8)

$$D(u, \theta, \omega, \tau_e) = \left[ 1 + \frac{1}{2} (\beta_{\perp}^i - \beta_{\parallel}^i) - u^2 \right] \cos^2 \theta - S(\omega, \tau_e) \sin^2 \theta \quad (5.1)$$

Assume that  $\tilde{\omega}$  and  $\tilde{\theta}$ , or  $\tilde{u}$  and  $\tilde{\theta}$  are an overstable solution of Eq. (5.1). Then

$$\begin{aligned} \text{Im} [D(\tilde{u}, \tilde{\theta}, \tilde{\omega}, \tau_e)] &= -\sin^2 \theta \left\{ \frac{1}{2} \frac{(\beta_{\perp}^i)^2}{\beta_{\parallel}^i} \text{Im} [Z_i'] \right. \\ &\quad \left. - \frac{\beta_e}{2} \text{Re} \left[ \zeta_v \left( 1 + \frac{6\Gamma_e}{\alpha} \right) \right] - \frac{\beta_e \beta_{\parallel}^i}{4} \text{Im} \left[ \frac{1}{\alpha} \frac{\left[ \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \right) \Gamma_e Z_i' + 2\Gamma_e \right] \left[ \alpha \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \right) Z_i' + 2\alpha \right]}{\beta_e \Gamma_e Z_i' - 2\beta_{\parallel}^i} \right] \right\} \end{aligned} \quad (5.2)$$

Now

$$\text{Sgn} [\text{Im} (Z_i')] = -\text{Sgn} (\tilde{u}),$$

$$\text{Sgn} [\zeta_v] = \text{Sgn} [\eta_{TC}] = \text{Sgn} (\tilde{u})$$

where

$$\text{Sgn} [x] = x / |x|,$$

so that

$$\begin{aligned} \text{Sgn} [\text{Im } D(\tilde{u}, \tilde{\theta}, \tilde{\omega}, \tau_e)] = \sin^2 \theta \left\{ \frac{1}{2} \frac{(\beta_{\perp}^i)^2}{\beta_{\parallel}^i} \left| \text{Im}(Z_i') \right| \right. \\ \left. + \frac{\beta_e}{2} \left| \zeta_v \left( 1 + \frac{\delta \Gamma_e}{\alpha} \right) \right| + \frac{\beta_e \beta_{\parallel}^i}{4} \left[ 8 \Gamma_e \beta_{\perp}^i + 4 \beta_e \Gamma_e^2 \right. \right. \\ \left. \left. + \beta_e \Gamma_e^2 \left( \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \right)^2 \left| Z_i' \right|^2 \right] \left| \text{Im } Z_i' \right| \right\} \text{Sgn}(\tilde{u}) \quad (5.3) \end{aligned}$$

Hence, if  $\sin \theta \neq 0$ ,  $\text{Im}(D)$  cannot vanish unless  $\tilde{u} = 0$ , so that Eq. (3.8) has no overstable solution unless  $\sin \theta = 0$ . If  $\sin \theta = 0$ , the dispersion relation reduces to Eq. (3.6).

The nonexistence of overstable solutions recovers the familiar fire-hose and mirror instabilities for collisional electrons (isotropic pressure tensor) and collisionless ions (anisotropic pressure tensor). The criteria of the instabilities in the present case can be expressed as follows:

$$\beta_{\perp}^i - \beta_{\parallel}^i < 2 \quad (5.4)$$

$$1 + \left( 1 - \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \right) \left[ \beta_{\perp}^i + \frac{\beta_e \beta_{\parallel}^i}{2} \frac{\left( 1 - \frac{\beta_{\perp}^i}{\beta_{\parallel}^i} \right)}{\beta_e + \beta_{\parallel}^i} \right] < 0 \quad (5.5)$$

Eq. (5.4) is the Alfvén fire-hose instability to which Eq. (3.6) is relevant, and Eq. (5.5) is the magneto-acoustic fire-hose instability to which Eq. (3.8) is relevant. In both cases, there is an angle  $\theta_0$  at which the stability is marginal. For the magneto-acoustic fire-hose instability, unstable waves propagate at angles  $\theta$  such that

$$0 \leq \theta < \theta_0 \quad \text{or} \quad \pi - \theta_0 < \theta \leq \pi, \quad (5.6)$$

and for the mirror instability, unstable waves propagate at angles  $\theta$  such that

$$\theta_o < \theta < \frac{\pi}{2} \quad \text{or} \quad \frac{\pi}{2} < \theta < \pi - \theta_o \quad (5.7)$$

The equation for  $\theta_o$  is just  $D(0, \theta_o, 0) = 0$ , which may be rewritten as

$$- \left[ 1 - \frac{1}{2} (\beta_{||}^i - \beta_{\perp}^i) \right] \cot^2 \theta_o = 1 + \left( 1 - \frac{\beta_{\perp}^i}{\beta_{||}^i} \right) \left[ \beta_{\perp}^i + \frac{\beta_e \beta_{||}^i}{2} \frac{\left( 1 - \frac{\beta_{\perp}^i}{\beta_{||}^i} \right)}{\beta_e + \beta_{||}^i} \right] \quad (5.8)$$

This angle  $\theta_o$  is real only if one of the instability criteria (5.4) and (5.5) holds.

### III-6 Numerical Solutions and Discussion

As has been shown in Section III, the dispersion relation which governs the behavior of magneto-acoustic waves is quite complicated. Unfortunately, analytic expressions for the damping rates are tractable only in very special limits. Therefore, we have resorted to numerical calculations to compare the damping rates of hydromagnetic waves for various values of the collision frequencies which characterize the altitude of the ionosphere.

We are interested in values of electron collision frequency appropriate to the ionosphere, at a height of 600 to 1000 Km. As pointed out earlier, that transition periods of electron collisions,  $\omega \tau_e$ , from the order of  $m_e/m_i$  to unity or less approximately correspond to the altitudes of the ionosphere for the present study. Furthermore, the present theory is of special interest because it clarifies both the characteristics of the upper F2 region which extends to the magneto-sphere and the transition between collisional and collisionless behavior of hydromagnetic waves. Therefore, we consider solutions of the dispersion relation over a range of  $\omega \tau_e$ , ranging from  $\omega \tau_e \sim m_e/m_i$  up to  $\omega \tau_e \sim 0$  (1) for low  $\beta$  ( $\approx 0.4$ ) plasma.

For the isotropic plasma ( $\langle T_{\parallel} \rangle \approx \langle T_{\perp} \rangle$ ),  $\omega$ , the numerical results of the dissipation rate as a function of the propagation angle  $\theta$  are obtained for higher ( $\omega \tau_e = 0.083$ ) and lower ( $\omega \tau_e = 0.46$ ), electron collision frequencies at  $\beta = 0.4$ . It shows (See Figure) the present results are similar to those obtained by Barnes (1966) for the collisionless plasma, therefore, we may conclude that the two peaks appearing in our calculation are due to ion Landau damping for  $\theta \lesssim 50^\circ$ , and electron thermal conduction and viscous effects for  $\theta > 50^\circ$ . Hence, the electron heating is always more important than the ion heating under the present conditions. This result explains the facts that the dissipation of hydromagnetic waves might play an important role in creating the phenomena  $\langle T_e \rangle > \langle T_i \rangle$ , in the upper F2 region, which agrees with observations (Comfort, 1970).



For the plasma with anisotropic ions,  $\langle T_{\parallel}^i \rangle \neq \langle T_{\perp}^i \rangle$ , the mirror and/or fire-hose instabilities might occur based on the instability criteria (5.4) and (5.5). Although the experimental data are still not enough to prove how far the anisotropic ions can satisfy the instability criteria, it is expected, however, that mirror and/or fire-hose instabilities might be relevant to irregular fluctuations or turbulent fields observed in the upper F2 region.

In summary, the present theory reveals the following properties of fast-mode magneto-acoustic waves under conditions expected in the upper F2 region: (1) Conductive damping is dominant at about 600 Km altitude and viscous damping is overwhelming at 1000 Km altitude; (2) Dissipation of hydromagnetic waves creates the bi-thermal phenomena ( $\langle T_e \rangle \gg \langle T_i \rangle$ ) which assures the propagation of ion-acoustic waves in the ionosphere; (3) The irregular or turbulent fields observed in the F2 region might be caused by the instabilities due to anisotropic ions (mirror and/or fire-hose instabilities); (4) Conductive and/or viscous dissipation is more effective than Landau damping due to wave-particle resonance interaction in the ionosphere, while Landau damping is the primary dissipation mechanism in the interplanetary space.

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## LIST OF FIGURE

Fig. III-1 Damping rate of magneto-acoustic mode as a function of propagation direction for higher ( $\omega \tau_e = 0.083$ ) and lower ( $\omega \tau_e = 0.146$ ) values of electron collision frequencies in an isotropic plasma ( $\langle T_{||} \rangle = \langle T_{\perp} \rangle$ ) with  $\beta \approx 0.4$ .

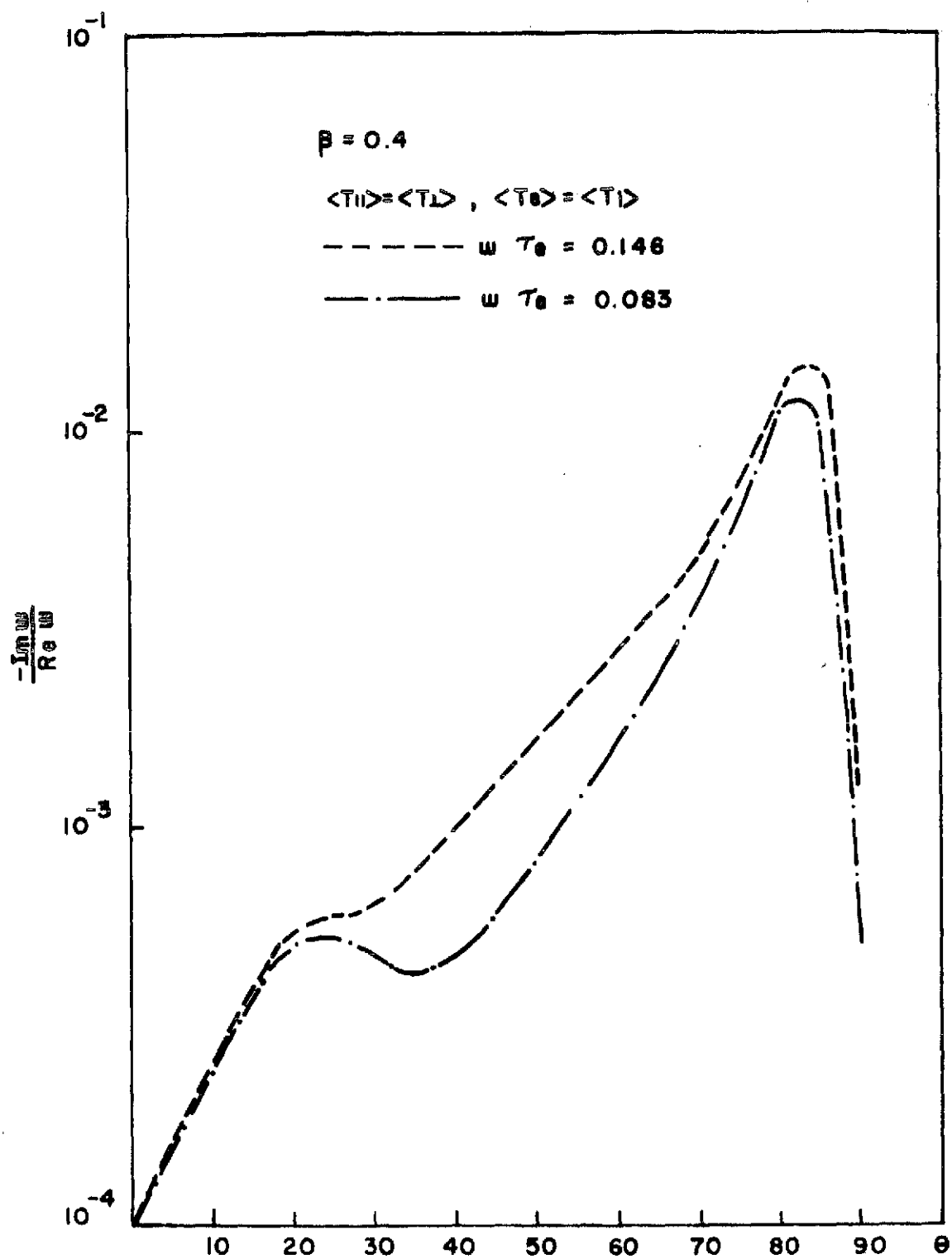


Figure III-1

## CHAPTER IV

### INTERPLANETARY SPACE ENVIRONMENT

#### IV-1 Introduction

It is well known that ordinary gasdynamic theory has been used successfully for a zeroth order theoretical continuum approach to the problem of solar wind interaction with Earth, and very possibly, Venus and Mars (Spreiter and Alksne, 1970; and Dryer, 1970). The interplanetary magnetic field was considered only in an implicit manner in the early solutions referenced in these papers. Explicit consideration of the field - recently completed by Shen (1972) and Hirsh and Reshotko (1971) - shows that the Lorentz force is effective only in the neighborhood of the subsolar region. The zeroth order approach has also been suggested (in varying degrees of quantitative analysis) for the case of moon by Lyon et al. (1967), Michel (1968), Wolf (1968), Siscoe et al. (1969) and Spreiter et al. (1970). The boundary condition on the sunlit hemisphere for these continuum studies was modified to permit absorption of all incident particles. An alternative approach for Moon has been used successfully by Whang (1968, 1969, 1970) who used a "guiding-center" model which reproduced experimental magnetic "anomalies" in the wake region (Ness et al. 1967; and Ness, 1972). This model (also with the complete absorption assumption) uses the microscopic kinetic equation to describe the motion of guiding centers along, but continuum flow perpendicular to, field lines. Whang and Ness (1970) further demonstrated the first experimental observation of standing magneto-acoustic waves which are inclined from the lunar limb in the direction of the "Mach cone." In order to interpret the data on a theoretical basis, they returned to the concept of the anisotropic propagation of magnetoacoustic waves as derived not only from the solution of the linearized continuum equations but also from the solution (Whang, 1970) of a guiding center plasma. This approach was formalized by Wang (1971) who used the same absorption boundary condition on the sunlit side of moon and found similar density profiles in the wake with a three-dimensional kinetic description. In effect, then, moon might almost be considered to be no more than a point disturbance in the solar wind. Yet, the realities of magnetic and plasma density anomalies discussed earlier

strongly suggest that moon and other "small" obstacles require further studies which start from first principles. For example, one might conceivably consider a boundary condition whereby particle reflection may be more specular-like, as in the case of an ionopause or magnetopause.

It has been suggested by Dryer (1970) that the obstacle's "Knudsen number",  $r_p/d$  (where  $r_p$  is the ambient proton's thermal gyroradius and  $d$  is the obstacle's physical or effective diameter) be used as a rough guide in choosing a basic approach to the interaction problem. That is, if  $r_p/d$  for an obstacle (a comet, say, at large A.U.) is large, say  $>10^3$ , we would likely use kinetic theory for the study of its interaction. Conversely, if  $r_p/d$  were  $<10^{-3}$ , we could proceed with great confidence with continuum theory. A "gray" area lies somewhere between these somewhat arbitrarily-chosen numbers. By analogy with hypersonic flight through the whole gamut of Earth-bound atmospheric and ionospheric levels, this "gray" area could be labeled as a "transition" region which conveniently covers our ignorance of the correct procedure for analysis. Alternatively, one might suggest the electron inertial length as the characteristic length of the ambient solar wind. The basis for this suggestion appears to be in its relation to shock wave thicknesses observed in the laboratory and in space. Another candidate is the proton gyroradius, based on the directed velocity, which is the characteristic thickness of the Earth's magnetopause. A related length scale could be the geometric mean of the proton and electron gyroradii. Detailed studies would require consideration of the boundary's nature: magnetic, nonmagnetic or absorbing.

We could, if desired, extend this approach beyond planets, comets, and asteroids (Greenstadt, 1971) to include our heliospheric motion through the interstellar medium. It was in the spirit of reducing the degree of random speculation to a more systematic theoretical approach that this idea was proposed. It is the purpose of this paper to continue in this spirit by providing an alternative

approach to our "shopping list" of theoretical approaches. Thus, we extend the well-known kinetic theory for particle interaction with a sphere (Karamcheti and Sentman, 1965; and Sentman and Karamcheti, 1969) to include the effect of an isotropic and anisotropic potential to represent the surfaces of some obstacles. The result may then be diffuse or even specular reflection. We are still interested, regardless of the outcome of the point regarding the boundary condition (on the sunlit side, say, of the moon) in the limiting case of perfect absorption (c.f., Wang, 1971). This can easily be specified by letting reflection be equal to zero. We will include it, however, in order to maintain generality. Thus the applicability of the boundary condition assumed herein to a specific obstacle (say, moon, Mercury, etc.) is not our immediate objective. It is our purpose to demonstrate a technique which, in principle, can be adjusted to include other boundary conditions and the interplanetary magnetic field.

#### IV-2 Discussions of Observations

Excluding ionospheric satellites from our discussion, the only potentially "small" obstacle for which observations exist is the Moon. Some recent experimental observations on and near moon may be relevant within the context of the fundamental kinetic approach which will be discussed in the next section. Other observations with respect to a possible limb shock (or Mach wave) as well as the magnetic wake anomalies have been extensively discussed in the literature (c.f., Ness, 1972; and Sonett and Mihalov, 1972) and, therefore, are not repeated here.

Figure 1 (based on Fig. 1 from Freeman, 1972) schematically shows the Moon's position with respect to a nominal position for Earth's shock wave on several dates: 13-15 December 1969 and 27 March 1970. On the latter date, Neugebauer et al. (1972) reported solar wind observations obtained with a proton spectrometer which was part of the ALSEP 12 instrument package on the lunar surface. As noted in the figure, the package was on the sunlit side of the Moon (with instrument normal directed about  $35^\circ$  to the east of the sun) and in front of the Earth's shock on 27 March 1970. Figure 2 (from Neugebauer et al., 1972) shows the velocity and density of the solar wind as measured on the surface of moon.

Vela 5 and Ogo 5 were also in front of the bow shock; the approximate position of the latter spacecraft is shown in Figure 1. Their measurements are also shown in Figure 2 in order to present "freestream" conditions. The time prior to 0711 UT is of interest here. (An interplanetary shock was measured at that time. The data subsequent to 0711 UT are of secondary interest here except for noting the related shock jump conditions at all three instruments. The time scales for these two spacecraft were shifted so as to coincide with ALSEP 12 at the shock wave.) Note that both spacecraft - in the ambient solar wind - measured velocities and densities which were, respectively, higher and lower than those measured on the lunar



surface. Neugebauer et al. (1972) suggest that this compression effect was caused by local lunar magnetic fields (c.f., Sonett and Mihalov, 1972). It is suggested, however, that the results discussed later in terms of density contours present a physically-plausible alternative explanation.

Several other experimental observations associated with the **moon** may also be relevant to the present study. Figure 1 also shows the Moon's approximate position on 13-15 December 1969 when the ALSEP 12 site was deep in the lunar night. Freeman (1972) has reported persistent observations of positive ion bursts, "probably of solar wind origin," when his suprathermal ion detectors at ALSEP 12 and 14 are located on the night side as indicated for the particular case of ALSEP 12 in Figure 1. The low flux suggested later by density contours and velocity vectors behind a typical spherical object may provide an explanation for these observations.

Another set of particle observations in front of the moon has been reported by Prakash (1970). Figure 3 shows contours of relative 2-5 keV electron flux on the sunlit side of the **moon** as measured by the MIT plasma cup on Explorer 35 during ~ 700 orbits (outside of the Earth's shock wave). These electrons, of course, represent only the high energy tail of the electron distribution function. Because of charge neutrality which exists in the solar wind, the data shown in Figure 3 must represent only the "tip of the iceberg" of an equally dense presence of protons in or near the same locations in front of the **moon**. The relationship of these preliminary observations to the density contours noted later suggests again that the kinetic approach may be physically plausible for the **moon**. If this suggestion is true, the presently-accepted concept of complete solar wind absorption on the sunlit side, with little or no "sputtering", may require **reexamination**.

### IV-3 Analysis

Let us consider a collisionless neutral plasma (i.e., solar wind) which flows past any "small" celestial object. On the surface of the object is an assumed angularly distributed positive electrostatic potential field. A schematic diagram of the problem is depicted in Fig. 4. This problem is equivalent to that of a free particle flow which interacts with a solid sphere with a body force due to the electric field on the surface. Hence, the mathematical model for the present problem can be described by the collisionless Boltzmann equation with the body acting as an external force on the gas, thereby eliminating the need for gas-solid surface boundary conditions. For simplicity, we shall consider steady-state ion motion within the solar wind; thus the governing equation for the present problem is

$$\underline{v} \cdot \frac{\partial f}{\partial \underline{r}} + \frac{\underline{F}}{m} \cdot \frac{\partial f}{\partial \underline{v}} = 0 \quad (1)$$

where  $f=f(\underline{r}, \underline{v})$  represents the singlet local velocity distribution function of the ions;  $\underline{v}$ , the ion velocity;  $\underline{F}$ , the external force given by

$$\underline{F} = - \text{grad } \Phi(\underline{r}) \quad (2)$$

where  $\Phi(\underline{r})$ , a scalar function of  $r$  and  $\theta$ , is the potential function due to the properties of the surface of the obstacle under consideration;  $m$ , the mass of an ion; and, finally,  $\underline{r}$  is the position vector.

The solution of the collisionless Boltzmann Equation has been discussed in detail by Karancheti and Sentman (1965) who direct attention to the advantages that result when solid surfaces are included in the kinetic problem as external forces. Thus, the velocity distribution function  $f(\underline{r}, \underline{v})$  to satisfy Eq. (1) for  $|\underline{r}| \rightarrow \infty$  can be written in the following form:

$$f(\underline{r}, \underline{v}) = A \exp \{ - B (\underline{v} - \underline{U}_\infty)^2 \} \quad (3)$$

with

$$\begin{aligned} A &= n_{\infty} (m/2\pi kT_{\infty})^{3/2} \\ B &= m/2kT_{\infty} \end{aligned} \quad (3a)$$

and where  $\underline{r}$  and  $\underline{v}$  are expressed in a spherical coordinate system.  $U_{\infty}$  represents the incoming solar wind velocity, and  $T_{\infty}$ , the ambient ion temperature.

The velocity distribution  $f(\underline{r}, \underline{v})$  is calculated numerically for each step. Initially, we assign values of the distribution function to the grid points in  $\underline{r}$ - $\underline{v}$  space. We convect the values of  $f$  according to Newton's law:

$$\left. \begin{aligned} \frac{df}{dt} &= 0 \\ \frac{d\underline{r}}{dt} &= \underline{v} \\ m \frac{d\underline{v}}{dt} &= -\text{grad } \Phi(\underline{r}) \end{aligned} \right\} \quad (4)$$

Thus, the distribution function can be calculated successively. It is convenient to nondimensionalize all the variables by measuring all the lengths, velocities and times in terms of the object's radius,  $a$ ; the thermal speed of the particle,  $c = 1/B^{1/2}$ ; and the time,  $aB^{1/2}$ , necessary to travel one "a" at the particle's thermal speed, respectively. Thus, we calculate the distribution function following the path given by Eq. (4). Thus, a non-spherically symmetric potential which represents the body surface can adequately be taken into account. It is important to note that the present solution is the lowest-order approximation to the solution of the Boltzmann equation and consequently, represents a process which ignores inter-particle collisions. Some additional details are given in the Appendix.

On the surface of the body, the velocity distribution function for the particles emitted from the surface is expressed by

$$f_{\text{emit}}(\underline{v}, a) = n(\theta_a) \left( \frac{B_w}{\pi} \right)^{3/2} \exp \left[ -B_w \underline{v}_a^2 \right],$$

where  $B_w = \left( \frac{m}{2kT_w} \right)$ ,  $\theta_a$  and  $v_a$  are the constants of integration which will be determined from the trajectory equation. In this study, the accommodation coefficient is assumed to be unity. More details concerning this point can be found from Prager and Rasmussen (1967).

Knowledge of the singlet local velocity distribution enables the calculation of the local density and velocity by taking moments with respect to the local velocity distribution.

#### IV-4 Method of Solution

Now, the task left to us is to calculate the velocity distribution using Eq. (4). A detailed discussion of this method is given by Prager and Rasmussen (1967). We shall not repeat it here; however, for completeness, we shall outline the method briefly as follows.

We prescribe a boundary condition for the distribution function at  $t = t_0$  in terms of some given function  $f_1$ , then,

$$f(\underline{r}(t_0), \underline{v}(t_0)) = f_1(\underline{r}(t_0), \underline{v}(t_0)). \quad (5)$$

Thus, the general solution can be expressed in terms of the function  $f_1$  by

$$f(\underline{r}(t), \underline{v}(t)) = f_1(\underline{r}_0(\underline{r}, \underline{v}), \underline{v}_0(\underline{r}, \underline{v})) \quad (6)$$

where

$$\begin{aligned}\underline{v}_0 &= \underline{v} + \frac{1}{m} \int_{t_0}^t \nabla \phi \, dt \\ \underline{r}_0 &= \underline{r} - \int_{t_0}^t \underline{v} \, dt\end{aligned}\quad (7)$$

which were obtained by integrating Eq. (4).

We may note that as  $f \rightarrow f_0$ , the initial value of  $f$  is recovered. The vectors  $\bar{v}_0$  and  $\bar{r}_0$  represent the six constants of integration required for the equation of motion. Therefore, the general solution is an arbitrary function of the constants of integration of the characteristics of Eq. (4). However, this arbitrary function is selected to agree with the boundary condition.

For the present problem, the spherical polar coordinate system for the physical space is chosen, then the solution to the collisionless Boltzmann equation corresponding to the condition Eq. (3) is given by

$$f(\underline{r}, \underline{v}) = A \exp \left[ -B(v_\infty^2(\underline{r}, \underline{v}) + U_\infty^2 + 2U_\infty \{v_{r_\infty}(\underline{r}, \underline{v}) \cos \theta_\infty(\underline{r}, \underline{v}) + v_{\theta_\infty}(\underline{r}, \underline{v}) \sin \theta_\infty(\underline{r}, \underline{v})\}) \right] \quad (8)$$

Now the task is to determine  $v_{r_\infty}(\underline{r}, \underline{v})$ ,  $v_{\theta_\infty}(\underline{r}, \underline{v})$ ,  $\cos \theta_\infty(\underline{r}, \underline{v})$ ,  $\theta_\infty(\underline{r}, \underline{v})$  and  $v_\infty(\underline{r}, \underline{v})$  by solving the equation of motion of a particle in a potential field  $\phi(r, \theta)$ .

After some mathematical manipulation, the distribution function for the particles coming from infinity and those emitted from the surface are now

$$f(r, v) = A \exp \left[ -B \left\{ v^2 + \frac{2e}{m} \Phi + U_\infty^2 - 2 U_\infty \sqrt{v^2 + \frac{2e}{m} \Phi} \cdot \cos \theta_\infty \right\} \right]$$

and

$$f_{\text{emit}}(v, r) = n(\theta_a) \left( \frac{m}{2\pi k T_w} \right)^{3/2} \exp \left[ -B_w \left( v^2 + \frac{2e}{m} \Phi \right) \right]. \quad (10)$$

The functions  $\cos \theta_\infty$  and  $\theta_a$  again will be determined from the trajectory equation of particles.

For convenience, we shall write the total local density in the form

$$n(r, \theta) = n_i(r, \theta) + n_{\text{emit}}(r, \theta) \quad (11)$$

where  $n_i(r, \theta)$  is the net contribution to the number density by the incident particles, i.e., the contribution of those particles that are moving toward the object with the effect of those particles passing by the object; and  $n_{\text{emit}}(r, \theta)$  is the contribution to the number density by the particles emitted from the surface with the temperature of the body surface,  $T_w$ . This is the effect of the boundary condition on the surface.

After some mathematical manipulation, we obtain

$$n_i(r, \theta) = \int_{-\infty}^0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( 1 + \frac{2e}{m} \frac{\phi(a, \theta)}{\chi^2} \right)^{1/2} f_i d\chi_r d\chi_\theta d\chi_\psi, \quad (12)$$

with  $\chi^2 = v^2 + \frac{2e}{m} \phi(r, \theta),$

where  $\vec{\chi} = (\chi_r, \chi_\theta, \chi_\psi); \quad \vec{v} = (v_r, v_\theta, v_\psi)$

and

$$f_i = n_\infty \left( \frac{B_\infty}{\pi} \right)^{3/2} \exp \left[ - S_\infty \left\{ \left( \frac{\chi}{U_\infty} \right)^2 + 1 - 2\chi_r \cos \theta + 2\chi_\theta \sin \theta \right\} \right] \quad (13)$$

with  $S_\infty = U_\infty / \left( \frac{2kT_\infty}{m} \right)^{1/2},$  the particle speed ratio.

$$n(r, \theta)_{emit} = n(\theta_a) \left( \frac{B_w}{\pi} \right)^{3/2} \int_0^\infty \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left( 1 + \frac{2e}{m} \frac{\phi(a, \theta)}{\chi^2} \right)^{1/2}$$

$$e^{-B_w \chi^2} d\chi_r d\chi_\theta d\chi_\psi \quad (14)$$

where  $n(\theta_a)$  will be determined by the assurance of continuity in the characteristic equation where  $n(\theta_a)$  will be found in terms of  $n_\infty$ .

The mean velocity field is determined by the first moment of the velocity distribution function,

$$\begin{aligned}
u_j(r, \theta) = & \frac{1}{n(r, \theta)} \left[ \int_{-\infty}^0 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_j \left( 1 + \frac{2e}{m} \frac{\phi(o, \theta)}{x^2} \right)^{1/2} \right. \\
& f_i(r, \psi) dx_r dx_\theta dx_\psi \\
& + \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x_j \left( 1 + \frac{2e}{m} \frac{\phi(o, \theta)}{x^2} \right)^{1/2} \\
& \left. f_{emit} dx_r dx_\theta dx_\psi \right] \quad (15)
\end{aligned}$$

with  $j = r, \theta$ .

Finally, the potential on the surface is derived from a charged particle interacting with an angularly-distributed charged particle layer on the surface and is given as follows:

$$\phi(r, \theta) = \int_{-\pi/2}^{\pi/2} \frac{Q (\cos \theta')^{1/2} d\theta'}{\left[ 1 + \left(\frac{r}{a}\right)^2 - 2 \left(\frac{r}{a}\right) \cos(\theta - \theta') \right]^{1/2}} \quad (16)$$

and/or

$$\phi(r) \sim \frac{Q'}{r^n}, \quad n = 2 \text{ and } 5 \text{ in the present study,} \quad (17)$$

where  $Q$  indicates a constant representing the product of quantum efficiency and subsolar potential and  $Q'$  is the usual force constant. A repulsive electric field of  $1 \text{ Vm}^{-1}$  (volt/m) at the subsolar point will be assumed for both asymmetric and spherically symmetric cases. In the latter case, of course,  $\phi$  is independent of  $\theta$  so that the two



cases are normalized at the subsolar point.

In the figures which are discussed below, the nondimensional radial distance,  $r/a$ , is denoted as  $R$ . Figure 5 shows the result of the integration of Eq. (16). It is seen that  $\Phi$  becomes essentially independent of  $\theta$  within several radii, thus, the main effect (of the potential which represents the force on the fluid) is confined to small values of  $R$  where a shock wave could eventually develop. In such a calculation, we have weighted the potential energy by the reflected particle's thermal energy,  $kT_w$ , as seen, for example, in Eq. (10).

#### IV-5 Numerical Results

A steady solar wind is assumed to be flowing past a "small" object with the following assumed parameters:  $U_\infty = 400 \text{ km sec}^{-1}$ ,  $n_\infty = 5 \text{ cm}^{-3}$ , and  $T = 10^5 \text{ }^\circ\text{K}$ . The particle speed ratio,  $S$ , is therefore 9.85. Other typical solar wind parameters will generally give  $S > 1$ . Sentman and Karamcheti (1969) show that the results for density contours are insensitive to variations in  $S$  once  $S > 1$ . We will assume that the present case, with the electric field, is similar and will not investigate this point any further in the belief that the present case will be sufficiently representative.

Using Equations (11) through (14) together with (4) through (10), the density distribution along the sun-obstacle axis (i.e.,  $\theta = 0^\circ$ ) is first found. Figure 6 shows  $n/n_\infty$  plotted versus the non-dimensional radius,  $R$ , in the absence of a field on the surface as well as with the presence of a field as discussed earlier. A sharp rise from the

ambient value of density to 2 and 3 times that value occurs within half of the obstacle's radial distance from the surface for the two cases, respectively. The results for the major portion of the affected flow field is shown by contours of constant density (with effective non-central force field) in Figure 7. It is seen that the surface density rapidly decreases as  $\theta$  moves toward the object's limb. A substantial paucity of particles is also seen in the immediate base region as shown also by Wang (1971) for the alternative case of absorption on the sunlit surface. Of particular interest, however, is the steep density gradient in the immediate vicinity of the limb. This result is, not unexpectedly, similar to that shown by Sentman and Karmacheti (1969) for the case of no field.

The density contours for  $S \approx 10$  (with and without a potential field) are also given by Dryer (1970). Also, the contours of constant density with effective central force field which are described by Eq. (17) are shown in Figs. 8 and 9 for  $n = 2$  and 5, respectively. Note that there is no substantive difference between the case of an asymmetrical force field (which represents a surface electric field as suggested for Moon by Walbridge (1968, 1972) and Grobman and Blank (1969) and a spherical inverse force field.

Finally, Equations 16 and 17 provide additional information about the flow field, namely, the streamline pattern. Figure 10 shows the results of the calculation for non-central force field where the length of the individual velocity vectors is proportional to their magnitude. Note, for reference, the length of the undisturbed

velocity vector at the left side of the figure. The expected deceleration of the flow on the sunlit side of the obstacle with subsequent acceleration in the expansion phase around the limb and in the wake is clearly seen. Mention has already been made above of the observation by Neugebauer et al. (1972) which may be relevant to the present study. It is seen that the flow, on the average, is similar to that for continuum flow around a sphere.

As noted earlier, the interplanetary magnetic field has been explicitly neglected. Further studies (with the present boundary condition) should, of course, take  $\underline{B}$  into account, but it is possible to make some qualitative observations regarding its effect on the flow. When  $\underline{v}$  is not parallel to  $\underline{B}$ , the induced electric field in the object's frame of reference is  $-\underline{v} \times \underline{B}$ . Thus, the Lorentz force is proportional to  $(\underline{v} \times \underline{B}) \times \underline{B}$ . We refer here to the volume current considered in MHD analyses and do not consider charge separation; thus, the  $\underline{E} \times \underline{B}$  drift can be neglected. Assuming, then, that  $\underline{B} = (B_r, 0, B_\varphi)$  in a spherical coordinate system whose axis is at the center of the sun, and  $\underline{v} = (u, 0, 0)$ , this force has the following components:  $(-u B_\varphi^2) \hat{r} + (u B_r B_\varphi) \hat{\varphi}$ . For the chosen undisturbed velocity components and "toward the sun" polarity of the field, the Lorentz force acts within the ecliptic plane to cause a distortion of the flow field. The density contours and velocity vector field (Figs. 7 through 10) would no longer be perfectly symmetrical. Wang (1971) has shown, for example, that the density distribution shows "complicated periodic structures" for cases of large gyroradii.

The direction of the force, indicated above for the example, will favor a higher density concentration on the sunward, morning quadrant of the flow field. The results presented here, then, must be considered to be a qualitative estimate of a self-consistent kinetic calculation which explicitly incorporates the effect of an arbitrarily-oriented, frozen-in, magnetic field. The skewing of the density contours (as well as other parameters) has been explicitly found in the continuum case by Shen (1972) and Hirsh and Reshotko (1971). Some of the latter's work is discussed by Dryer (1970). An additional manifestation of this effect would be the unsymmetrical "Mach cone" which has been observed by Whang and Ness (1970) in the case of moon and which is anticipated by the limb density gradient in Figs. 7, 8, and 9 for the zeroth order solution.

#### IV-6 Enclosure

Sentman and Karamcheti (1969) have pointed out that the spacing of the constant density contours in front of the sphere is relatively insensitive to variations in the speed ratio,  $S$ , once  $S > 1$ . Behind the sphere, the contours (hence the wake) elongate while there is a decrease of the width of the compression and the "expansion fan" which extend rearward from the limb. Although results (which include effects of the electrostatic potential) for  $S$  different from 10 are not presented here, we believe that they would produce similar effects as noted above. It is seen that the ridge of increased density ( $n > n_0$ ) - due to reflected protons - extends downstream in the direction of what might be called a developing "Mach cone." The

results suggest, for high Knudsen numbers, that effective collisional encounters would first become important along this ridge of high density gradient. Sentman and Karamcheti (1969) have suggested that " ... perhaps it is the high-density ridge that develops into a shock wave as collisions become important." Also, the shock may indeed form in a high Knudsen flow, but it would probably first appear downstream from the body and, as Sentman and Karamcheti point out: " ... the stagnation streamline would probably be the last region in the flow to become aware of the presence of the shock"! Flybys of objects (such as Mercury and Pluto which are potential candidates for high Knudsen number flow) should then be scheduled for near-limb passes with a data accumulation rate high enough to detect possible density gradients suggested by Figs. 7, 8, and 9. A suggestion by Barnes et al. (1971) is possibly related to this discussion. These authors suggest that limb shocks can be caused by localized magnetic anomalies intrinsic to the moon. They further suggest that small length scales (geometric mean of electron and proton gyroradii) are expected to promote instabilities which are, in turn, induced by wave-particle interactions. Pertinent to the present study is the possibility that anomalous transport properties (i.e., "collisions") would be generated.

Equally important would be detection of streamline patterns suggested by Fig. 10. This figure indicates that there could be a sudden deceleration along the subsolar streamlines within a distance of  $\sim 1$  radius from the surface. Scattering at higher energy densities

occurs near the surface in the direction of the on-coming flow, but the protons are quickly turned and deflected around the limb. It is surprising to note that the dark side of the object is impacted by a very low, but finite, density of particles. The observations of Neugebauer et al. (1972) and Freeman (1972) noted above may, again, be relevant within the present context.

We also point out that the present kinetic approach to the interaction of the solar wind with "small" obstacles (i.e., those celestial objects characterized by large Knudsen numbers) is suggested as an alternative to continuum and quasi-kinetic techniques. Indeed the results found in the investigation suggest that, for a given proton thermal gyroradius, a shock wave would develop first at the limb, gradually building up - as we choose larger obstacles - to a "conventional" detached bow shock as in Earth's case. Alternatively, for a given "small" obstacle, the effect would be the same if the solar wind were to get much hotter so that the gyroradius were to decrease gradually by several orders of magnitude.

We state this important point in an alternate way, that is, by discussing an increase in the obstacle's characteristic dimension in a slightly different way. The increase in size could certainly be gained by added material (i.e., choosing a larger planet), but it could also be achieved by invoking the presence of an atmosphere, ionosphere, and/or a magnetosphere. The physical mechanism for new boundary conditions (ionopause, magnetopause, or even surface penetration) is a condition which could be incorporated in related

studies. For example, surface absorption on the sunlit hemisphere, as in the case of the moon, could be incorporated in the expression for the number density of particles which are reflected from the surface (Eq. 14).

Finally, we should note that Wang (1971) has considered the magnetic field effect in his calculation of solar wind-moon interaction; however, the condition of a completely absorbing surface is used. The reader is reminded that the magnetic field is not explicitly taken into account in the present study; although an estimate of its effect suggests that introduction of the Lorentz force would be confined to a skewing of the density contours and an associated distortion of the flow pattern. Also, provision is not made in the present work for particle absorption (with no subsequent reemission) in the sunlit hemisphere.

Inasmuch as several statements have been made regarding the development of shock waves, "Mach cones," and the like, provided the characteristic fluid scale length is small relative to the obstacle's scale length, a final comment is made to relate the present kinetic approach to its extreme "competitor": continuum theory. It is well known that the "gain and loss" term in the kinetic description is given by the collision integral. For free particle flow, this term is zero because there are no collisions of any kind. Similarly, in continuum theory - at least at the level of Euler's approximation regarding the absence of transport properties such as viscosity - the collision integral is again found to be zero.

because the distribution function is found to be Maxwellian. There is, then, no contradiction in the statement that kinetic theory, as discussed herein, preserves the characteristics of the continuum theory.



## APPENDIX

We have employed the finite difference technique for obtaining numerical results. We have chosen that "i" be the label for the r - coordinate and "j" be the label for the  $\theta$ -coordinate, starting clockwise with intervals of  $15^\circ$  in a range  $0 \leq \theta \leq 180^\circ$ . Also we have set up the origin of the coordinate system at the center of the body, therefore the local velocity distribution is expressed as  $f_{i,j}(r_i, \theta_j, v_r^{i,j}, v_\theta^{i,j}, v_\phi^{i,j})$ . We have assumed that the local velocity distribution is Maxwellian for  $R = 10$  (which is effectively  $R = \infty$ ) and  $0 \leq \theta \leq 180^\circ$ , then we compute each point successively by using Eq. (1) and Eq. (2). Newton's equations of motion (i.e., Equation (4) is used here to track the particle trajectory due to the effects of the non-central force potential field. Because of the non-straight line characteristics, we can only fix our initial points on the boundary; the subsequent points, as said, were determined by Newton's equations of motion. However, our grid points are fixed points, therefore, a Range-Kutta subroutine is used for the interpolation.

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## LIST OF FIGURES

1. Schematic sketch of Moon's position during sunlit (27 March 1970) and night-side (13-15 December 1969) surface observations. (Based on Fig. 1 from Freeman, 1972).
2. Solar wind (proton) velocity and density observations in the free stream (Vela 5 and Ogo 5) compared with lunar surface observations (AISEP 12). (Neugebauer et al., 1972).
3. Relative electron flux contours in front of the Moon as found by Explorer 35 (Prakash, 1970).
4. Schematic sketch of the interaction of the solar wind with a spherical obstacle. The characteristic radius of the obstacle,  $a$ , may include an atmosphere, ionosphere, and/or a magnetosphere. The obstacle is assumed to be "small" when the characteristic solar wind length (possibly the proton thermal gyroradius) is equal to or greater than, say,  $10^{-1} a$ .
5. Dimensionless electrostatic potential,  $\phi^*$ , in the vicinity of a spherical obstacle.  $R = r/a$ . The potential,  $\phi$ , is normalized by the quantity  $Q$  which includes the surface properties and quantum efficiency of the photoionization process (Walbridge, 1972).
6. Proton number density, normalized to undisturbed solar wind density, along the obstacle - Sun axis ( $\theta = 0^\circ$ ).  $S = U_\infty / (2 kT/m)^{1/2} = 9.85$ . Results are shown for an angularly-distributed surface electric potential. The surface repulsive electric field at the subsolar point is taken to be  $1 \text{ Vm}^{-1}$ . The result for no electric surface potential is essentially the same as that found by Sentman and Karamcheti (1969) for  $S = 10$ .
7. Constant density contours with non-central force field potential for  $S = 9.85$  in the vicinity of a "small" spherical obstacle. Surface repulsive electric field at the subsolar point is  $1 \text{ Vm}^{-1}$  and decreases as  $\cos^2 \theta$ . Note development of a continuum-like "Mach cone" at the limb, compression near the subsolar point, and expansion immediately beyond the limb.
8. Constant density contours with central-force-field potential for  $n = 2$  and  $S = 9.85$  in the vicinity of a "small" spherical obstacle. Surface electric field at the subsolar point is  $1 \text{ Vm}^{-1}$ , i.e., the same as for the non-central field in order to make a direct comparison with that case.

9. Constant density contours with central-force-field potential for  $n = 5$  and  $S = 9.85$ .
10. Velocity vectors (or "streamline" pattern) with non-central-force field potential for  $S = 9.85$  in the vicinity of a "small" spherical obstacle. Surface electric field at the subsolar point is  $1 \text{ Vm}^{-1}$ . Note turning of the flow on the sunlit hemisphere and acceleration of particles around the limb as in continuum flow.

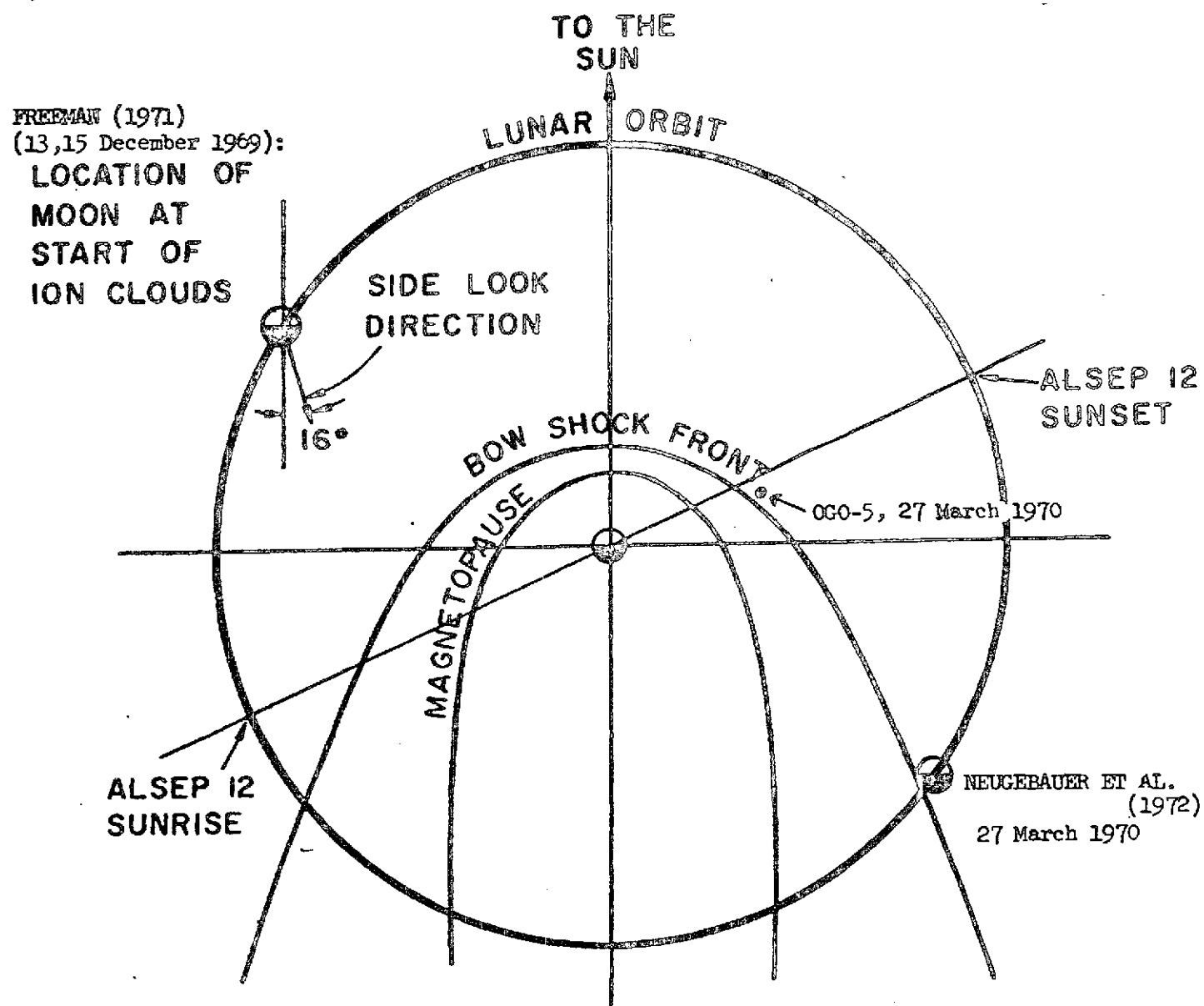
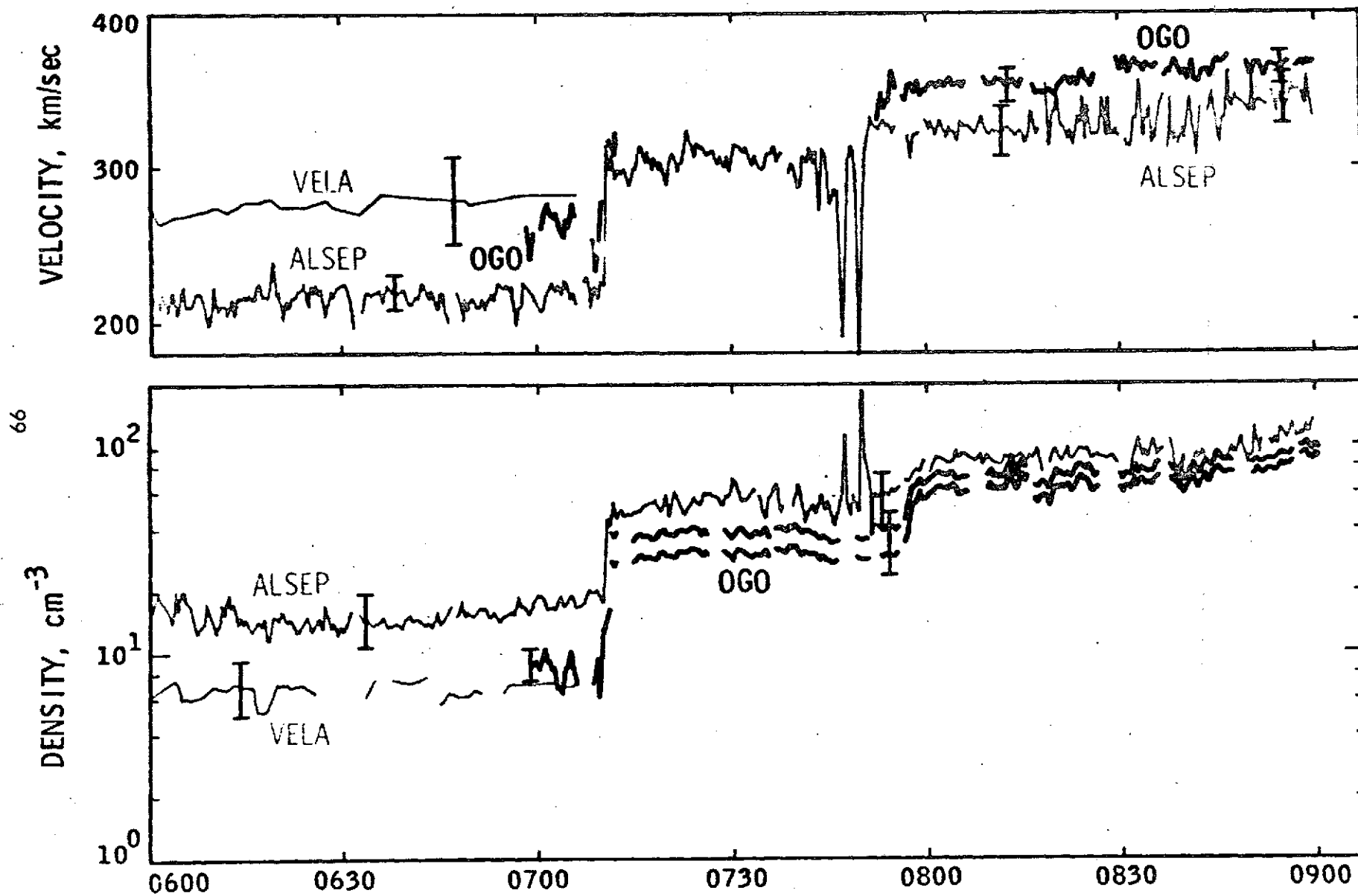


Fig. IV-1



UT March 27, 1970

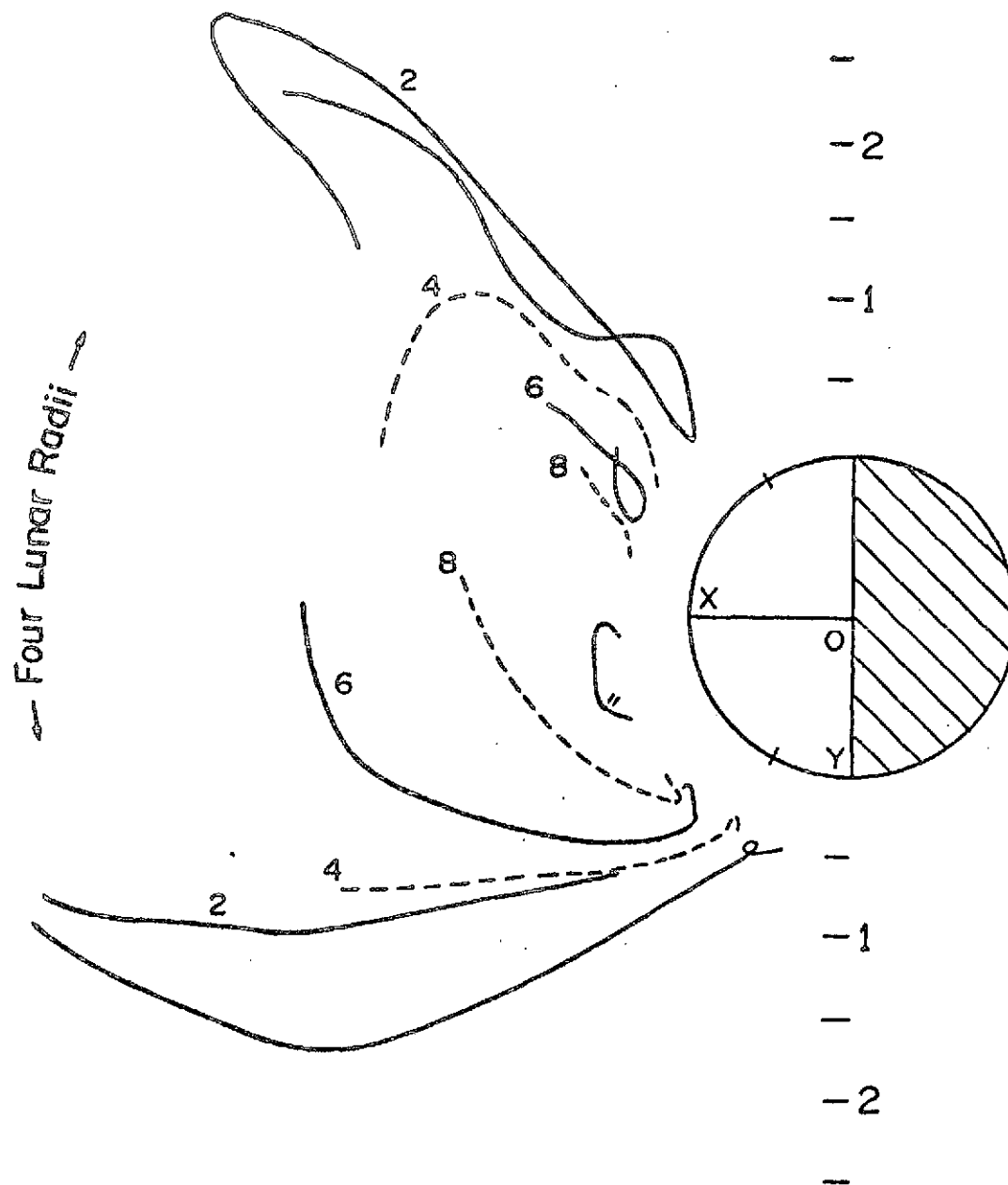
Fig. IV-2

-45° -50° -60°  
-40°

PRAKASH [1971]

Explorer 35 (1967-68; 700 orbits)

Relative Electron Flux (2-5 keV)



40° 45° 50° 60°

Fig. IV-3  
100



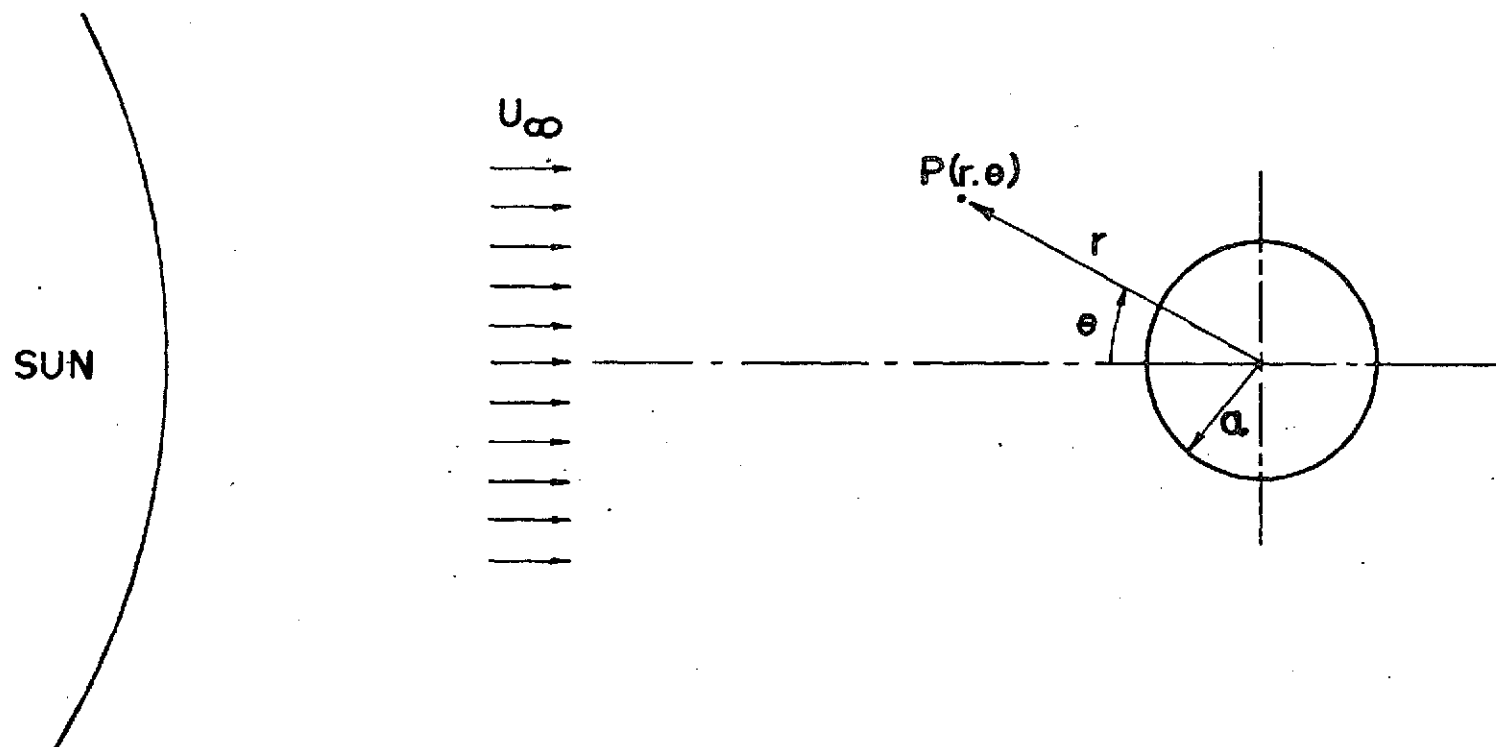


Fig. IV-4

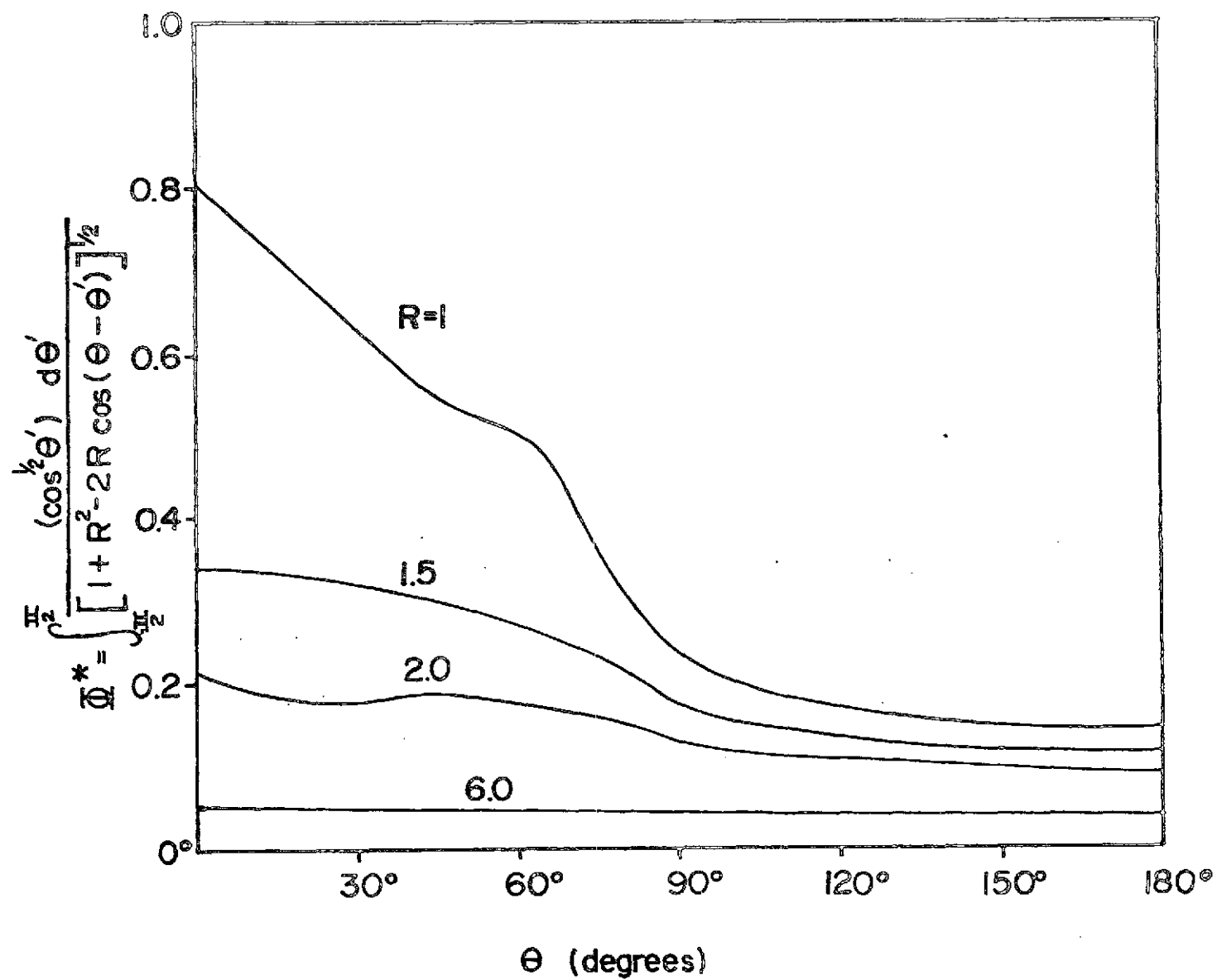


Fig. IV-5

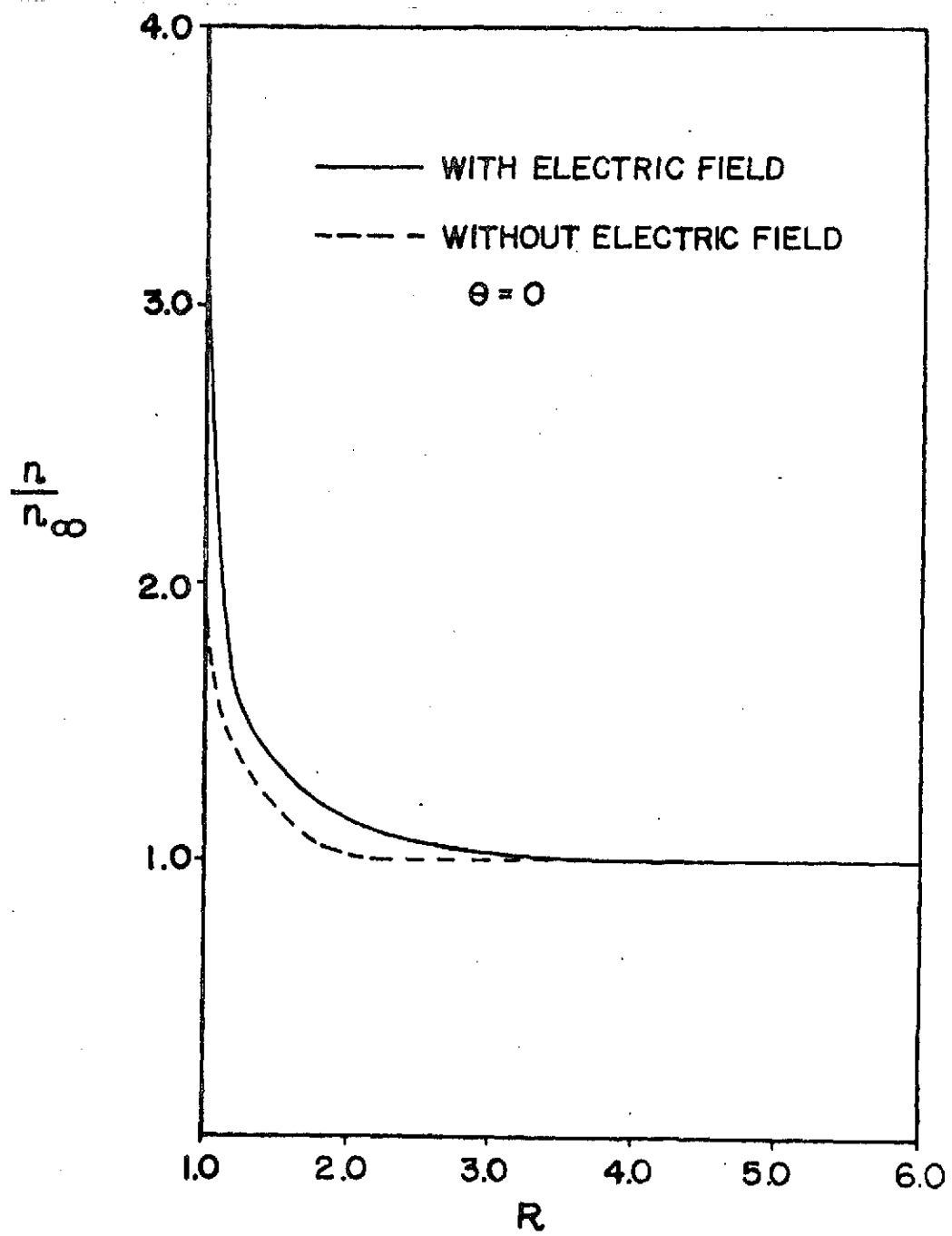


Fig. IV-6

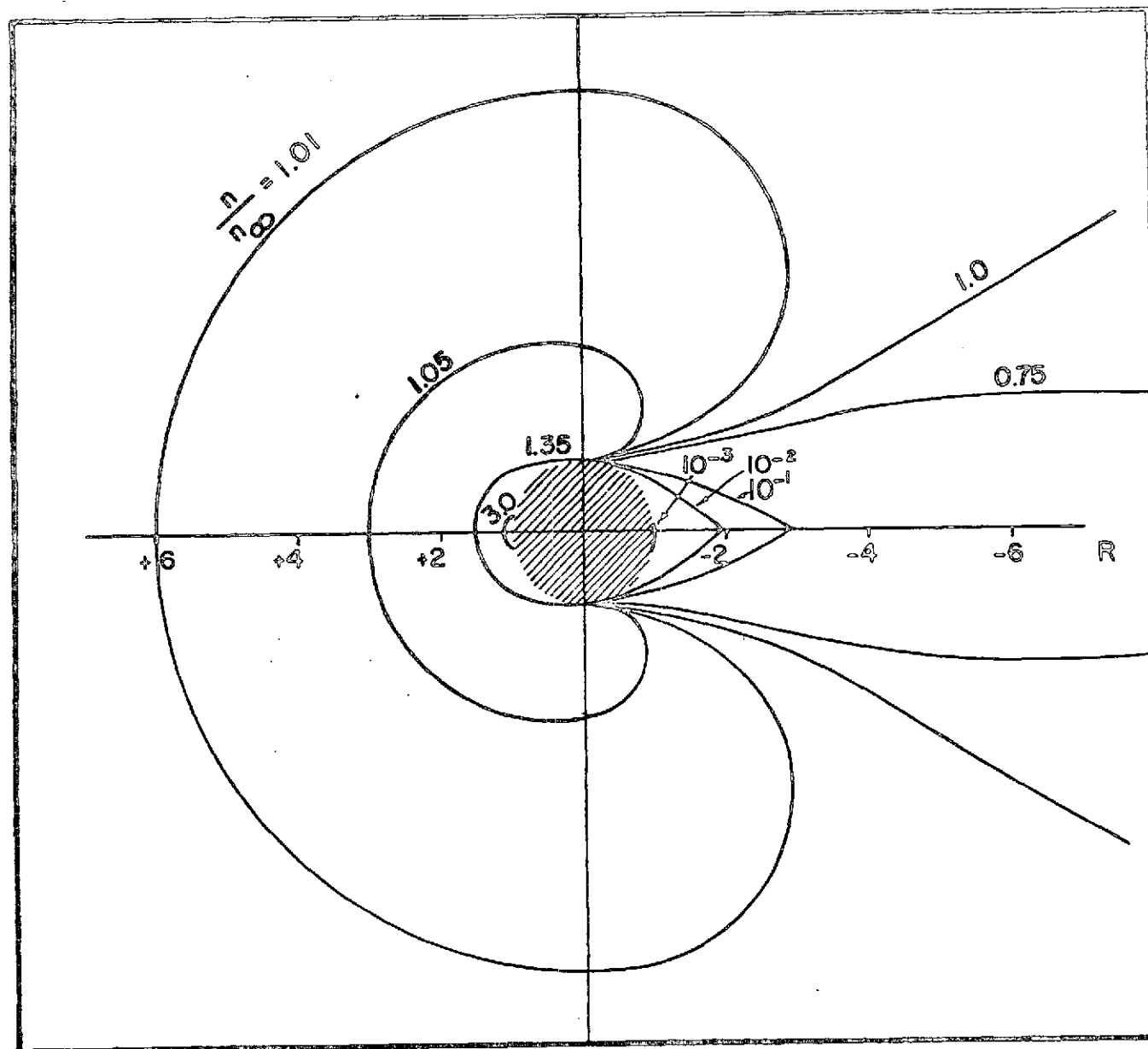


Fig. IV-7

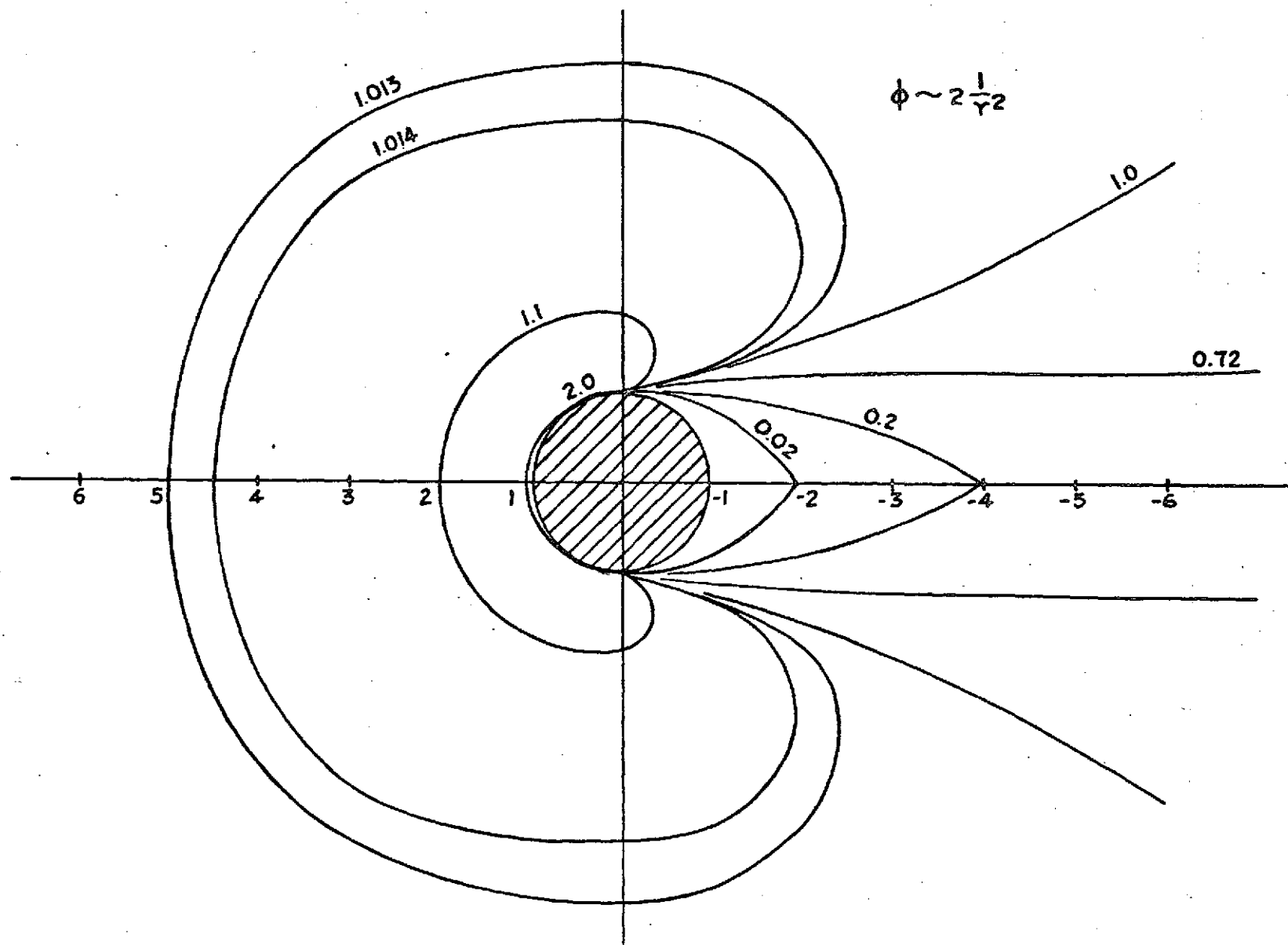


Fig. IV-8

$27 = 5^-$

106

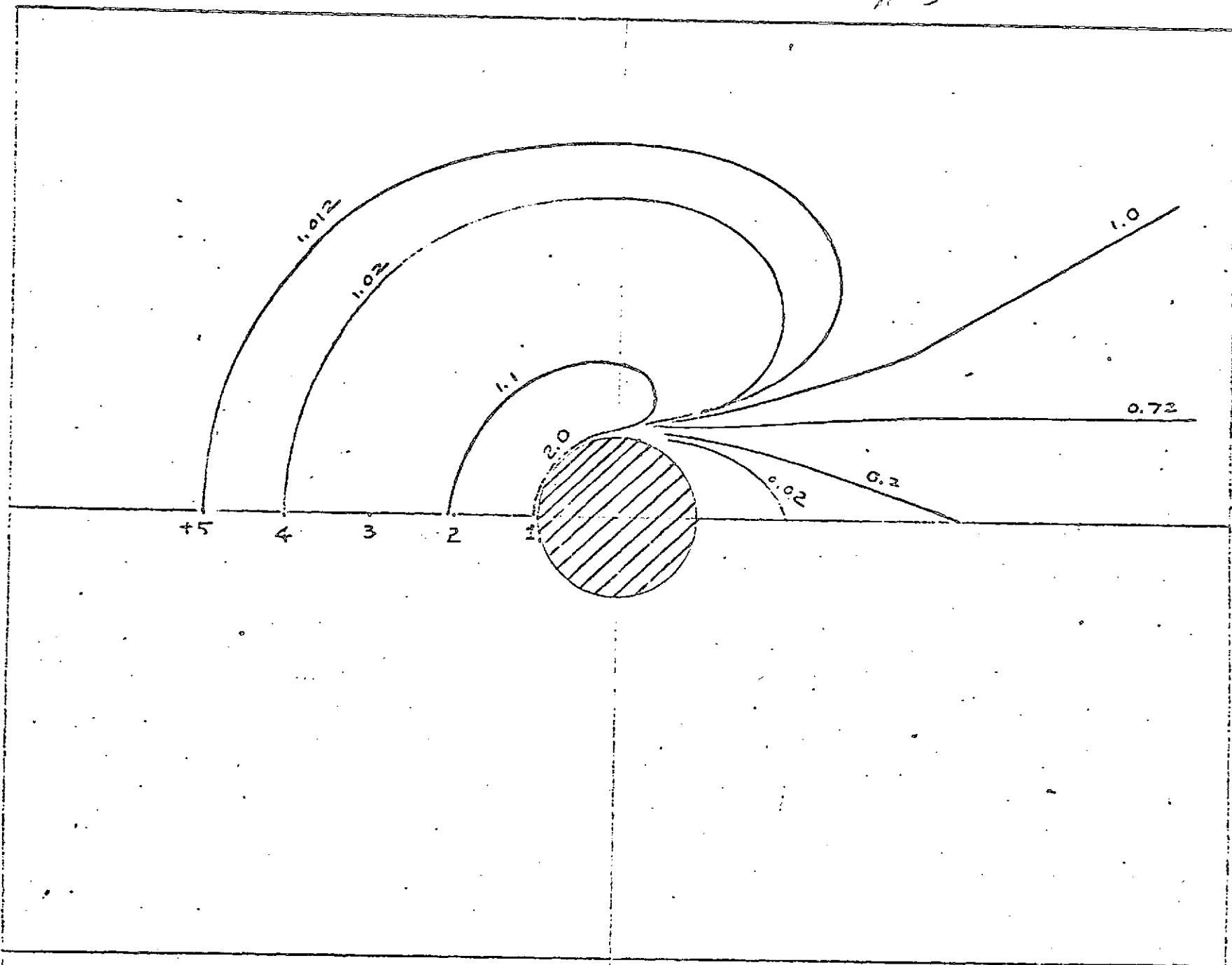


Fig. IV-9

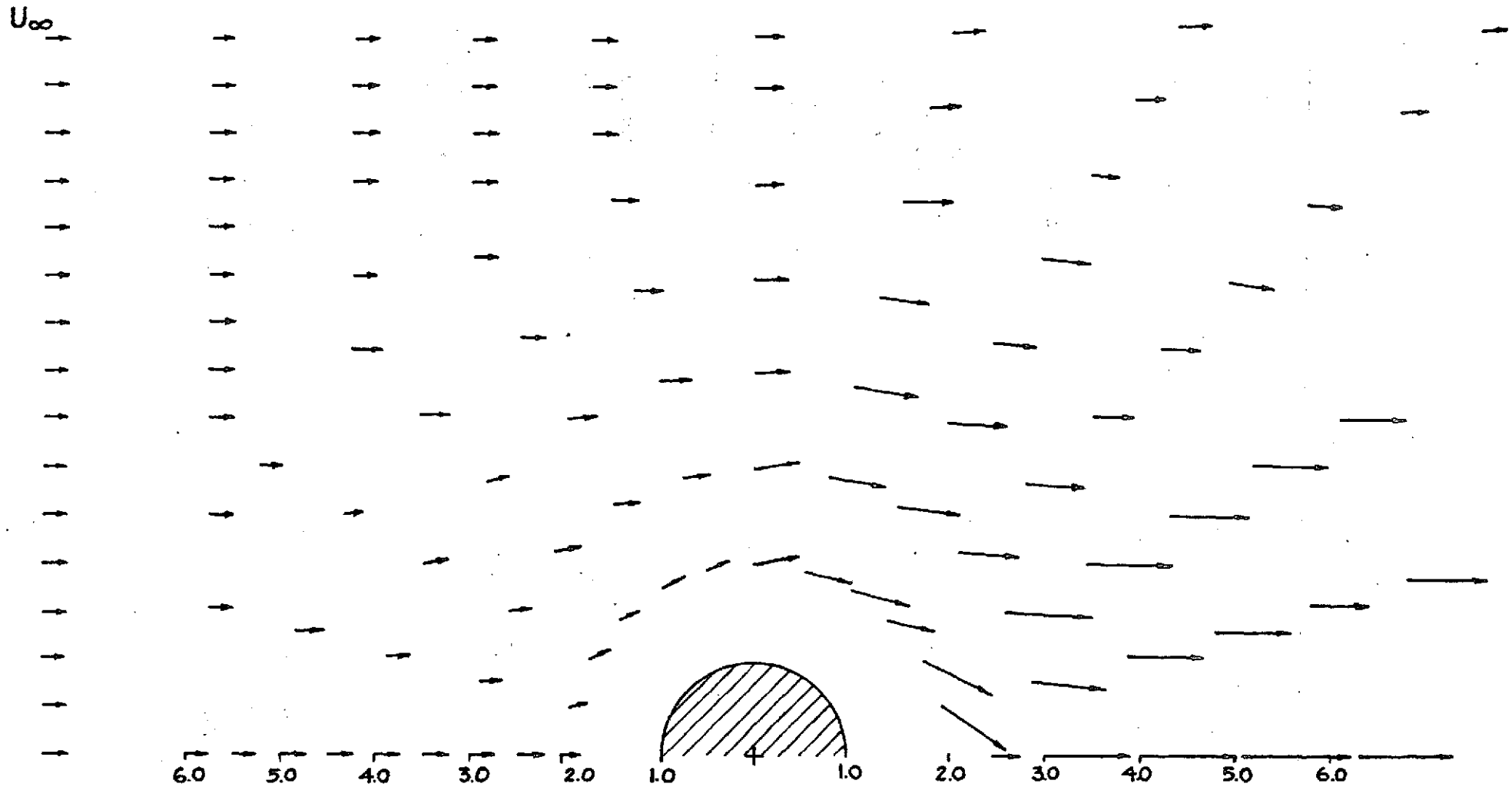


Fig. IV-10

## CHAPTER V

### CONCLUSIONS AND RECOMMENDATIONS

During this study we investigated dynamical problems ranging from the earth's upper atmosphere to interplanetary space. The theories used to construct the models to seek the solutions for these problems also covered a wide range; from continuum theory of magnetohydrodynamics to discrete motion of particles, i.e., kinetic theory of plasmas.

It is not possible to obtain a complete solution due to the complexity of the mathematical model of nature of the physical problems which we have attacked. Therefore, a number of appropriate assumptions are introduced.

However, the results which we have obtained are reasonably accurate when compared with the observations. There is still room for improvement of our models. Outlined briefly are possible future efforts.

(1) The dynamical model for the responses of the neutral atmosphere is a very important subject in understanding the characteristics of the atmosphere from 90 Km to 200 Km. In the present study, we treat the atmosphere as a mixture of gases. The effects on each component of the constituent gases in the mixture is simply ignored. In order to understand the energy budget and concentration of the atmosphere, it is necessary to examine the dynamical equations for each constituent gas. This problem should be examined carefully.

(2) Concerning the propagation of hydromagnetic waves in the ionosphere, only one particular case has been examined as discussed in Chapter III. Other important waves such as ULF, ELF and VLF need to be investigated.

(3) Finally, the subject we have touched is the solar wind interaction with planetary objects. We have neglected the interactions between ions and electrons, and ignored the wake and sheath structure. Therefore, if possible, these problems deserve particular attention in our future Spacelab program.



## **APPENDIX A**

**List of Lectures Held on the Campus of The University of  
Alabama in Huntsville Resulting from this Contract**



The University  
Of Alabama  
In Huntsville

P.O. Box 1247  
Huntsville, Alabama 35807

Announcement of Lecture

THE RESTLESS GEOMAGNETIC FIELD

Speaker: Dr. Wallace H. Campbell  
Time: 10:30 a.m., Thursday, 20 December 1973  
Place: The University of Alabama in Huntsville  
Research Institute Building, Room M-50  
Sparkman Drive, Huntsville, Alabama

Abstract

A review of the geomagnetic field changes occurring with periods between 0.5 sec and 1 month which reflect the dynamic behavior of our space environment.

Biographical Information

Currently Geophysicist, Geological Survey, Department of the Interior, Dr. Campbell received the B.S. (Louisiana State University, 1950), the M.A. (Vanderbilt University, 1953), and the Ph.D. degrees (University of California at Los Angeles, 1959). He has taught at various institutions, lectured widely and published many scientific papers. Among his committee assignments have been the American Geophysical Union, the International Association of Geomagnetism and Aeronomy, International Scientific Radio Union, and Inter-Union Commission on Solar Terrestrial Physics. Dr. Campbell has organized many of the national and international meetings he has attended as a delegate.

For further information, call Dr. S. T. Wu at 825-6413.



The University  
Of Alabama  
In Huntsville

P.O. Box 1247  
Huntsville, Alabama 35807

Announcement of Lecture

THE REMOTE SENSING OF STRATOSPHERIC AEROSOLS FROM SATELLITES

Speaker: Dr. Fred L. Bartman

Time: 10:30 a.m., Monday, 19 November 1973

Place: The University of Alabama in Huntsville  
Room M-50, Research Institute  
Sparkman Drive, Huntsville, Alabama

Abstract

Members of the High Altitude Engineering Laboratory of the University of Michigan are studying the feasibility of determining stratospheric concentrations of minor constituents and pollutants from satellite measurements of infrared absorption during solar occultation. As part of this study the extinction of simplified models of stratospheric aerosols has been examined to determine: 1) How they might interfere with the measurements of molecular constituents, and 2) Whether continuous monitoring of stratospheric aerosols may be carried out by satellite solar occultation measurements. The results of this study of aerosols are discussed after the aerosol model and its optical characteristics are described in detail.

Biographical Sketch

Dr. Bartman received the Ph.D. from the University of Michigan, where he is currently Director of the High Altitude Engineering Laboratory and Professor in the Departments of Aerospace Engineering and Atmospheric and Oceanic Science. An expert in high altitude atmospheric sciences, he has published numerous papers and served as a consultant to many different governmental agencies.

For further information, call Dr. S. T. Wu, 895-6413.



University of Alabama  
in Huntsville

P.O. Box 1247  
Huntsville, Alabama 35807

### Announcement of Lecture

## IONOSPHERIC STORMS IN LOW LATITUDES AND INTERPLANETARY SECTOR FIELD EFFECTS ON THE POLAR CAP

Speaker: Dr. S. Matsushita  
Time: 10:30 a.m., Friday, 9 February 1973  
Place: The University of Alabama in Huntsville  
Research Institute, Room M-50  
Sparkman Drive  
Huntsville AL

### Abstract

The ionization density increase in the F2 layer, on the average, over the magnetic equator during geomagnetic storms has long been a puzzle. This is explained by a decrease in the fountain effect mainly caused by a change of the E-W electrostatic field (hence the vertical plasma drift).

It has recently been reported by several scientists that the interplanetary magnetic sector fields cause the polar cap geomagnetic field changes. This phenomenon is re-examined in detail and is explained by a shift of the  $Sq^P$  current pattern.

### Biographical Information

Currently Senior Scientist, High Altitude Observatory, National Center for Atmospheric Research, Professor of Astro-Geophysics, Univ. of Colorado, and consultant to NOAA, Dr. Matsushita holds the M.Sc. and the Dr. Sc. (Kyoto Univ.). He was a lecturer at Kyoto Univ. and a member of the research staff, University College, London, before joining the staff of the Univ. of Colorado in 1955. He is the recipient of the Tanakadate Award, Society of Terrestrial Magnetism and Electricity, 1950, and the Scientist Award, Science Research Society of America, 1963. Dr. Matsushita is co-author of two books, has been editor of journals and proceedings, and published over 120 papers. Principal research has been on solar terrestrial relationship and physical behavior ionosphere, magnetosphere and the earth's magnetic field. He discovered lunar effects on lower ionosphere and the special ionospheric zone of sporadic E over the magnetic equator.

For further information, please call Dr. S. T. Wu - 895-6413

Announcement of Lecture

SOME RECENT ADVANCES IN TIME SERIES ANALYSIS

Speaker: Dr. Emanuel Parzen  
Chairman, Department of Statistics  
State University of New York at Buffalo

Time: 10:30 a.m., Friday, 3 November 1972

Place: The University of Alabama in Huntsville  
Research Institute, Room M-50  
301 Sparkman Drive, N.W.  
Huntsville, Alabama

Abstract

This survey of current research will include a discussion of: estimation of covariances and inverse covariances of stationary normal time series; autoregressive spectral estimation; efficient closed form estimation of the parameters of moving average schemes and mixed autoregressive moving average schemes; and estimation of parameters of models of signal plus noise.

Biographical Information

Dr. Parzen received the B.A. (1949) from Harvard, the M.A. (1951) and the Ph.D. (1953) degrees from The University of California at Berkeley. Among the positions he has held are Research Scientist, Hudson Labs., Columbia University; Guest Professor of Mathematics, Imperial College, London; Visiting Professor of Statistics, MIT; Professor of Statistics, Stanford University; and a Fellow, IBM Systems Research Institute, New York City, before becoming Professor and Chairman, Department of Statistics, State University of New York at Buffalo. Dr. Parzen holds membership in numerous associations and has been referee for many journals. Associate Editor, SIAM Control Journal and Editor, Time Series Analysis Series of Holden Day, he has authored over 35 papers and, among others, the following books: Modern Probability Theory and its Applications, Stochastic Processes and Time Series Analysis Papers.

For further information, call: Dr. S. T. Wu, 895-6413

Announcement of Lecture

## DISSIPATION OF HYDROMAGNETIC WAVES

Speaker: Dr. R. J. Hung

Time: 10:30 a.m., Monday, 19 June 1972

Place: The University of Alabama in Huntsville  
Research Institute Conference Room  
Sparkman Drive  
Huntsville, Alabama 35807

Abstract

Propagation of hydromagnetic waves is modified by transport phenomena due to Coulomb collisions. Collisional effects can vary from region to region in a plasma of interest. In particular, waves of a given period may see one region of a plasma as collisionless (in the sense that the wave period is short compared with the Coulomb collision time) and another region of the plasma as collision-dominated. For example, a transition from collisional to collisionless behavior may take place for a wave propagating upward in the ionosphere or solar corona, because the Coulomb collision time decreases with altitude. It is therefore of considerable interest to investigate how hydromagnetic waves behave in the whole plasma, including the collisional, intermediate and collisionless regions, and in particular, how the waves are dissipated. The work reported here is concerned with the dissipation of hydromagnetic waves, especially in the transitional outer solar corona. The theory is equally applicable to the ionosphere and other areas of space physical and astrophysical problems.

Biographical Information

Dr. Hung received the B.S. degree in 1957 (National Taiwan University), the M.S. in 1966 (University of Osaka) the Ph.D. in 1970 (The University of Michigan). A National Research Council Postdoctoral research associate, he is currently working in the Theoretical Studies Branch of Ames Research Center. With his background in plasma physics magnetohydrodynamics and fluid mechanics, he has authored various papers and received several honors and awards.

For further information, please call Dr. S.T. Wu - 895-6413



Announcement of Lecture

STUDIES OF ATOMIC COLLISIONS IN THE LABORATORY  
AND IN THE IONOSPHERE

Speaker: Dr. M. A. Biondi

Time: 10:30 a.m., Friday, 19 May 1972

Place: The University of Alabama in Huntsville  
Research Institute, Room M-50  
Sparkman Drive  
Huntsville, Alabama 35807

Abstract

The path connecting laboratory studies of relevant atomic collision processes with models of ionospheric behavior is illustrated using dissociative electron-ion recombination and charge transfer as examples. Microwave and optical interferometer techniques are applied in laboratory studies to determine the nature of the electron-ion recombination process and its dependence on electron energy. Drift tube - mass spectrometer techniques are used in the study of low energy ion-molecule reactions such as non-resonant charge transfer. The results of these laboratory studies are applied to a model of ionospheric behavior under normal and artificially perturbed conditions. The predictions of the model are compared with interferometric studies of 6300A OI nightglow emission line profiles and with photometric studies of 6300A intensity changes produced by powerful ground-based transmitters.

Biographical Information

Dr. Biondi received the Ph. D. in physics in 1949 from the Massachusetts Institute of Technology. From 1949-52 he was resident physicist, later, Manager of the Physics Department, Westinghouse Research Laboratory. Currently Professor of Physics at the University of Pittsburgh, he has been consultant to various agencies, including NSF and NRL, and published numerous papers. Areas of interest are atomic physics, interactions and reactions involving electrons, ions and excited atoms, solid state physics, electromagnetic properties of metals at liquid helium temperatures.

For further information, please call Dr. S. T. Wu - 895-6413

Announcement Lecture

INTERACTION OF ELECTRONS WITH  
WATER VAPOR WITH ATMOSPHERIC APPLICATIONS

Speaker: Dr. John J. Olivero, Jr.  
Postdoctoral Research Associate-Instructor  
(Physics and Astronomy)  
University of Florida

Time: 10:30 a.m., Friday, April 14, 1972

Place: The University of Alabama in Huntsville  
Research Institute Conference Room  
Sparkman Drive  
Huntsville, Alabama

Abstract

We examine the consequences of electron impact on water vapor in terms of the microscopic details of excitation, dissociation, ionization, and combinations of these processes. Basic electron impact cross section data are assembled in many forms and incorporated into semianalytic functions suitable for analysis with digital computers. Energy deposition in water vapor is discussed and the energy loss function ( $2 dE/dx$ ) is presented along with the "eV per ion pair" and the efficiencies of energy loss in various processes. We consider several applications of electron-water vapor interactions in the atmospheric sciences. We discuss  $H_2O$  comets, aurora and airglow, and lightning. In general we find that other processes are the more likely source of  $H^*$ ,  $O^*$ , and  $OH^*$  emissions with the probable exception of lightning.

Biographical Information

Currently Postdoctoral Research Associate-Instructor (Physics and Astronomy), University of Florida, Dr. Olivero holds the M. S. degree from College of William and Mary and received his Ph.D. from University of Michigan in 1970. He has been employed by NASA-Langley Research Center in Virginia and the University of Michigan. Dr. Olivero is the author of several publications.

For further information, call Dr. S. T. Wu at 895-6413.





Announcement of Lecture

THE NEGATIVE BINOMIAL DISTRIBUTION: DIFFERENT POINTS OF VIEW

Speaker: Dr. G. P. Patil

Time: 10:30 a.m., Monday, 27 March 1972

Place: The University of Alabama in Huntsville  
Research Institute, Room M-50  
Sparkman Drive  
Huntsville, Alabama 35807

Abstract

The negative binomial distribution has been studied extensively in the past, and it has been found that data arising in various situations in many fields fits this distribution. Naturally, then, one wonders about the underlying chance mechanisms, and it turns out that there are several such mechanisms which have no apparent relationship. The emphasis of this paper is on bringing together in a single notation stochastic models generating the negative binomial distribution (NCBD), and the basic character of the paper is that of a systematic review and exposition of the available results on the subject. A few new results and comments given will, hopefully, clarify the structure of this important distribution. With the exception of the important  $\alpha$ -truncated negative binomial distribution, distributions formed by randomly stopped sums, mixtures and other modifications of the negative binomial distribution are not discussed.

Biographical Information

Dr. Patil received the M.Sc. (1955) from Poona University and the M.S. and Ph.D. (1959) from the University of Michigan. He has served on the faculties of the universities of Michigan, McGill and Wisconsin and presently is Professor of Mathematical Statistics at Pennsylvania State University. He has been consultant with USDA, Forest Service, NIH, HEW, and the Laboratory of Applied Studies. He is the author or co-author of "Dictionary and Bibliography of Discrete Distributions," some 50 research publications and editor of "Random Counts in Scientific Work" and "Statistical Ecology" and director of several international symposia.

For further information, please call Dr. S. T. Wu - 895-6413



Announcement of Lecture

MEASURING WAVES, ESTIMATING WAVE SPECTRA AND SPECIFYING AND FORECASTING WAVES  
GIVEN THE WIND FIELD AND WIND FIELD FORECASTS

Speaker: Dr. Willard J. Pierson  
Time: 10:30 a.m., Thursday, 9 March 1972  
Place: The University of Alabama in Huntsville  
Research Institute, Room M-50  
Sparkman Drive  
Huntsville, Alabama

Abstract

Waves have been measured by the Tucker Shipborne wave recorder and by stereophotogrammetric, buoy and airborne laser techniques. From these measurements, the spectra of the gravity wave components for so-called "fully developed seas," for growing seas as a function of fetch and duration, and for dead seas and swell have been found. From these data, and from theories on wave generation, non-linear interactions, wave dissipation and wave propagation, a computer based procedure for specifying and forecasting waves on the open ocean in terms of their spectra, given wind fields and wind field forecasts, has been developed. Such computed spectra can be used to determine the wave climate for a given area of the ocean and to predict how ships, towed objects, such as spent rockets cyclinders, and buoys would behave in the waves.

Biographical Information

Professor Pierson received the B.S. degree from the University of Chicago in 1944 and the Ph.D. degree from New York University in 1949, when he came a member of the faculty; he is Professor of Oceanography currently. Editor of the Journal of Atmospheric Science, he is a member of the American Geophysics Union (vice president, section on oceanography 1965-57), American Meteorology Society, American Astronautical Society, Marine Technology Society and Sigma Xi. He has published two books and numerous articles. Principal research interests include motion of ships in waves, wave analysis by time series, and forecasting by computer techniques, use of radar on spacecraft to study oceans, turbulence, ocean currents.

For further information, please call Dr. S. T. Wu - 895-6413

Announcement of Lecture

DAILY VARIATION OF THE IONOSPHERIC F2 EQUATORIAL  
ANOMALY IN THE AMERICAN AND ASIAN SECTORS

Speaker: Dr. David N. Anderson  
CIRES Visiting Fellow  
University of Colorado

Time: 10:30 a.m., Thursday, December 16, 1971

Place: The University of Alabama in Huntsville  
Research Institute, Room M-50  
Sparkman Drive  
Huntsville, Alabama

Abstract

When observed noontime values of the maximum electron density in the ionospheric F2 region are plotted as a function of magnetic latitude, a curve is produced which has two peaks, one on either side of the dip equator at  $\pm 16^\circ$  dip latitude. The present work mainly will discuss the theoretical investigations of the daily variation of this latitude distribution in NMAX(F2) (the so-called Appleton or equatorial anomaly) and specifically attempts to account for the longitudinal differences observed between the American and Asian sectors. Other investigations include the effects of the neutral wind in producing north-south asymmetry in the anomaly and the solar cycle effect on the anomaly's daily variation.

Biographical Information

Currently CIRES Visiting Fellow at the University of Colorado, Boulder, Dr. Anderson holds the M.S. degree from North Carolina State University and received his Ph.D. from University of Colorado in 1971. He has been employed by Los Alamos Scientific Laboratory in New Mexico, NCAR and Dept. of Commerce in Boulder, Colorado. Member of American Geophysical Union, Phi Kappa Phi, NDEA Fellowship ('62-'64) North Carolina State, HAO Graduate Assistantship, University of Colorado. He has published some technical papers.

For further information, call Dr. S. T. Wu at 895-6413

Announcement of Lecture

MAGNETISM OF THE EARTH IN SPACE

Speaker: Dr. S. Matsushita  
Time: 2 p.m., Thursday, 18 November 1971  
Place: The University of Alabama in Huntsville  
Research Institute, Room M-50  
Sparkman Drive  
Huntsville, Alabama

Abstract

Behavior of the earth's ionosphere and magnetosphere estimated from the viewpoint of geomagnetic variation fields are reviewed. Key observations needed to resolve the remaining theoretical questions concerning magnetically quiet conditions are discussed. Concerning magnetic disturbances, the current state-of-the-art and a new idea based on recently available data are presented, including suggestions to find missing links.

Biographical Information

Currently Senior Scientist, High Altitude Observatory, National Center for Atmospheric Research, Professor of Astro-Geophysics, Univ. of Colorado, and consultant to NOAA, Dr. Matsushita holds the M.Sc and the Dr. Sc. (Kyota Univ.). He was a lecturer at Kyota Univ. and a member of the research staff, University College, London, before joining the staff of the Univ. of Colorado in 1955. He is the recipient of the Tanakadate Award, Society of Terrestrial Magnetism and Electricity, 1950, and the Scientist Award, Science Research Society of America, 1963. Dr. Matsushita is co-author of two books, has been editor of journals and proceedings, and published over 120 papers. Principal research has been on solar terrestrial relationship and physical behavior ionosphere, magnetosphere and the earth's magnetic field. He discovered lunar effects on lower ionosphere and the special ionospheric zone of sporadic E over the magnetic equator.

For further information, please call Dr. S. T. Wu - 895-6413

Announcement of Lecture

DYNAMIC EFFECT OF SUDDEN HEATING IN THE AURORAL ATMOSPHERE

Speaker	Dr. Paul B. Hays
Time:	10 a.m., Friday, 22 October 1971
Place:	The University of Alabama in Huntsville Graduate Studies Building, Room 110-111 Sparkman Drive Huntsville, Alabama

Abstract

The auroral atmosphere is, during times of magnetic storms, subjected to intense local heating. This energy addition results both from particle impact and electric fields; however, the electric field heating appears to be dominant. The neutral atmosphere responds to local heating by setting up cellular dynamic systems which cause changes in thermospheric composition. These composition changes have been detected and are compared with theoretical predictions.

Biographical Information

Currently Professor of Aerospace Engineering at the University of Michigan, from which he received the Ph.D. degree, Dr. Hays did post doctoral work at Queens University, Belfast. A leading authority in atmospheric science, Dr. Hays has published numerous papers and books in his field.

For further information: Call Dr. S. T. Wu - 895-6000

Announcement of Lecture

SPECTRAL MOMENT ESTIMATION

Speaker: Dr. Kenneth S. Miller  
Time: 10 a.m., Thursday, 13 May 1971  
Place: The University of Alabama in Huntsville  
Research Institute Lecture Room, M-50  
301 Sparkman Drive  
Huntsville, Alabama

Abstract

The problem to be discussed is estimating the moments of the spectral density of a complex stationary process. This problem is of interest in such diverse fields as tropospheric communication channels, turbulent velocity phenomena (ionized wakes) and the estimation of arterial blood velocities using Doppler techniques.

Some of the details have appeared in published papers. This brief lecture will outline the general philosophy governing the choice of suitable estimators, explore some of the ramifications of the problem, and indicate how to compute the statistics of various estimators.

Biographical Information

Dr. Kenneth S. Miller, a native New Yorker, received the B.S. degree in chemical engineering and the M.A. and Ph.D. degrees in mathematics from Columbia University. Post-doctoral work was done at the Institute for Advanced Study, Princeton, in 1950.

A radar officer in the U. S. Navy during World War II, Dr. Miller was, successively, instructor, assistant professor, associate professor and Professor of Mathematics at New York University. During this time he also acted as a consultant to various industrial and governmental agencies on problems associated with system analysis, noise, mathematical machines, and applied mathematics. He is the author or co-author of over fifty research papers and fifteen books. Currently he is at the Riverside Research Institute, New York, New York, and as Adjunct Professor of Mathematics at Fordham University.

Dr. Miller is a member of Sigma Xi, Tau Beta Pi, Pi Mu Epsilon, the American Mathematical Society, and a senior member of the IEEE.

For further information: Call Dr. S. T. Wu - 895-6413



Announcement of Lecture

CHEMISTRY OF THE UPPER ATMOSPHERE  
AND THE ROLE OF TRANSPORT PROCESSES

Speaker: Dr. Richard A. Craig  
Time: 10 a.m., Tuesday, March 9, 1971  
Place: The University of Alabama in Huntsville  
Research Institute Lecture Room, M-50  
301 Sparkman Drive  
Huntsville, Alabama

Abstract

The important chemical reaction affecting the neutral composition of the upper stratosphere, mesosphere, and lower thermosphere will be discussed. The role of atmospheric transport processes will be emphasized and the problem of taking these into account elucidated.

Biographical Information

Dr. Craig received the A.B. degree in astronomy from Harvard in 1942, the M.S. and Sc. D. both in meteorology from M.I.T., in 1944 and 1948 respectively. Currently Professor and Chairman, Department of Meteorology at Florida State University, Dr. Craig served as a research associate at Woods Hole Oceanographic Institution, a research fellow at Harvard College Observatory, and a research meteorologist and branch chief at the Air Force Cambridge Research Center. Active in professional societies, Dr. Craig has been associate editor of the Journal of Meteorology and the Journal of Geophysical Research. He has been a consultant to the National Science Foundation, White Sands Missile Range and NASA. He has published widely in various journals, co-authored a number of books and authored The Edge of Space: Exploring the Upper Atmosphere (Doubleday).

For further information: Call Dr. S. T. Wu 895-6413

Announcement of Lecture

SOME ASPECTS OF ATMOSPHERIC WAVES IN REALISTIC ATMOSPHERES

Speaker:

Dr. Richard Seigmund Lindzen

Time:

10 a.m., Tuesday, 2 February 1971

Place:

The University of Alabama in Huntsville  
Research Institute Lecture Room M-50  
301 Sparkman Drive, N.W.  
Huntsville, Alabama

Abstract

The theoretical behavior of tides and gravity waves will be discussed, accounting for the effects of viscosity, thermal conductivity and mean shears. Special emphasis will be given to the semi-individual tide and to atmospheric free oscillations.

Biographical Information

Dr. Lindzen received the B.A. degree in physics (1960), the M.S. (1961) and the Ph.D. (1964) in applied mathematics from Harvard. The following year he was appointed a research associate at the University of Washington and, in 1965, a NATO post doctoral fellow at the Institute for Theoretical Meteorology at the University of Oslo. He received appointments at the National Center for Atmospheric Research as a research scientist from 1966 to the end of 1967, returning as a lecturer the following summer. He has been a visiting lecturer at the University of California at Los Angeles, a visiting professor at Tel Aviv University, and Associate Professor of Meteorology at the University of Chicago since 1968.

A member of various organizations and committees, Dr. Lindzen is the recipient of the N.C.A.R. Outstanding Publication Award, the A.M.S. Meisinger and A.G.U. Macelwane Awards and an Alfred P. Sloan Fellowship. He has published extensively, both in the United States and in foreign journals.

For further information: Call Dr. S. T. Wu, 895-6413



Announcement of LectureTHE STABLE AURORAL RED ARC AND ITS  
INTERACTION WITH THE NEUTRAL ATMOSPHERE

Speaker: Dr. R. G. Roble

Time: 10 a.m., Tuesday, December 1, 1970

Place: The University of Alabama in Huntsville  
Research Institute Lecture Room M-50  
301 Sparkman Drive, N.W.  
Huntsville, Alabama

Abstract

The stable auroral red arc which occurs in subauroral latitudes during geomagnetic storms is examined in the light of recent observations. The various excitation mechanisms which have been postulated to explain the red arc are examined critically. Satellite observations of electron temperature and the topside electron density structure are used to calculate a red arc considering thermal conduction in the electron gas as the only energy source. The calculated  $\lambda$  6300 emission rate, geographical position, and horizontal extent are compared with photometric data obtained from airglow observatories and the results show good agreement. The calculated neutral gas temperature response to electron heating within the arc is shown to be small in agreement with  $\lambda$  6300 doppler temperature measurements made with a Fabry-Perot interferometer on the red arc of October 31/November 1, 1968.

Biographical Information

Raymond G. Roble received B.S.E. degrees in Engineering Physics and Engineering Mathematics (1957), the M.S.E. degree in Mechanical Engineering (1961), and the Ph.D. in Aeronomy (1969), University of Michigan. After serving three years with the U. S. Navy and later working with Bendix Research Laboratories, Dr. Roble returned to the University of Michigan High Altitude Engineering Laboratory. He designed and built the Airglow Observatory, which has a 6" diameter Fabry-Perot interferometer and an airglow photometer, used to study the stable auroral red arc. Currently Dr. Roble is an atmospheric scientist at the National Center for Atmospheric Research, Boulder, Colorado.

For further information: Call Dr. S. T. Wu, 895-6000



Announcement of Lecture

"THE COMPOSITION OF UPPER ATMOSPHERE"

Speaker:

Professor Alfred Nier

Time:

10 a.m., Tuesday, November 13, 1970 \*

Place:

The University of Alabama in Huntsville  
Research Institute Lecture Room M-50  
301 Sparkman Drive, N.W.  
Huntsville, Alabama

Abstract

Mass spectrometers, carried on sounding rockets and satellites, have been successfully employed for studying the neutral and ionic composition of the earth's atmosphere about 100 km. The measurement problem is complicated by the high speed of the vehicles, but methods have been developed for minimizing the difficulties. Above 100 km neutral  $N_2$ ,  $O_2$ ,  $O$  and  $Ar$  are in diffusive equilibrium. Determinations of absolute atomic oxygen densities are uncertain because of the high reactivity of this constituent.

Helium is an extremely interesting constituent. In the lower thermosphere above White Sands, New Mexico, the winter concentrations exceed the summer by a factor of 10. Also, helium does not appear to be in diffusive equilibrium. In the polar region there is no evidence for a departure from diffusive equilibrium.

Biographical Information

Born in St. Paul, Minnesota, 1911, Dr. Nier holds the degrees of B.A. (1931) and M. S. (1933) in electrical engineering and Ph.D. (1936) in physics from the University of Minnesota. Currently Dr. Nier is Regents' Professor of Physics at the University of Minnesota, where he successively was Assistant Professor (1938-40), Associate Professor (1940-44), Professor (since 1944), and Chairman, School of Physics (1953-65). From 1936-38, he was a National Research Fellow, Harvard University. On leave from 1943-45, Dr. Nier was a physicist in charge of analytical instrument development laboratory for Kellogg Corporation, a part of the Manhattan District program.

\*Originally announced date: 23 October 1970

For further information: Call Dr. S. T. Wu, 895-6413

Announcement of Lecture**"STRUCTURE AND VARIATIONS OF THE HETEROSPHERE"**

Speaker: Dr. Luigi G. Jacchia

Time: 10 a.m., 29 October 1970

Place: The University of Alabama in Huntsville  
Research Institute Lecture Room M-50  
301 Sparkman Drive N.W.  
Huntsville, Alabama

Abstract

Atmospheric composition is kept nearly homogeneous by mixing to a height of about 90 km (the homosphere); above 100 km we have diffusive separation of the atmospheric constituents (the heterosphere). The heterosphere is dominated by two energy sources: solar EUV and the solar wind; the manifold, large variations that are observed in this region can all be related, directly or indirectly, to variations in the intrinsic intensity of these two sources and to variations in the location of the sources relative to the observer. Atmospheric models can be successfully constructed, in which the temperature and density variations are empirically related to solar and geomagnetic parameters. Some of the more recent findings concerning the structure and variations in the heterosphere will be presented.

Biographical Information

Born in Trieste, Italy, 1910, Dr. Jacchia received the Ph.D. degree from the University of Bologna in 1932. Currently a Physicist at the Smithsonian Astrophysical Observatory, he was a D.I.C. staff member (1943-49) and a Research Associate in Electrical Engineering (1949-53), at M.I.T. From 1941-45, Dr. Jacchia did defense work, foreign language broadcasting and monitoring and was a Scientific Consultant to the Office of War Information. Previously, he was a Research Associate at Harvard College Observatory, held a fellowship of the Society for the Protection of Science and Learning for Research, University of London Observatory, was a lecturer, University of Bologna, and an astronomer, University of Bologna Observatory.

For further information: Call Dr. S. T. Wu, 895-6413.

Announcement of Lecture

"UPPER ATMOSPHERIC SPECTROSCOPY"

Speaker: Dr. A. E. S. Green

Time: 10 a.m., Thursday, October 15, 1970

Place: The University of Alabama in Huntsville  
Research Institute Lecture Room M-50  
301 Sparkman Drive, N.W.  
Huntsville, Alabama

Abstract

Spectroscopy and atomic physics are so intimately entwined with planetary aeronomy that progress in this geophysical-astronomical discipline in large measure is dependent upon progress in certain phases of these basic disciplines. In this lecture, we illustrate the interdependence by a discussion of photon, electron and proton stimulation of dayglow and auroral spectra.

Biographical Information

Dr. Green received the B.S. from the City College of New York (1940), the M.S. from the California Institute of Technology (1941), and the Ph.D. degree from the University of Cincinnati (1948), all in physics. Manager of Space Science Laboratory, General Dynamics (1959-63), Dr. Green has held academic appointments at the University of Cincinnati as Associate Professor (until 1953), at Florida State University as Professor and Scientific Director, Tandem Van deGraaff Program (until 1959), and at the University of Florida as Graduate Research Professor (since 1963); he served as Operations Analyst and Physicist with the U. S. Air Force (1944-45). Dr. Green held various visiting appointments including Oak Ridge National Laboratory, Los Alamos, Aerospace Corporation, and Stanford Linear Accelerator.

For further information: Call Dr. S. T. Wu, 895-6413.

RESEARCH INSTITUTE  
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

Announcement of Lecture

"UPPER ATMOSPHERIC RESPONSE TO TRANSIENT HEATING"

Speaker: Dr. Gary E. Thomas  
Time: 10:30 A.M., Friday, September 25, 1970  
Place: Research Institute, The University of Alabama  
in Huntsville, Lecture Room M-50, 301  
Sparkman Drive, N. W., Huntsville, Ala.

Abstract

The time-dependent response of the upper atmosphere to transient heat sources is considered. The basic problem is that of heating of a compressible, heat-conducting fluid, which is described in the one-dimensional case by an analytic solution. Comparisons with satellite drag data of such first-order solutions are shown to be useful in determining energy requirements and in determining some constraints on the spatial distribution of the heating. Recent OGO-6 mass spectrometer and interferometric temperature measurements show that atmospheric disturbances during geomagnetic storms are much more prominent in the auroral zones. These results suggest that joule dissipation of auroral currents are important contributors to orbital perturbations of satellites.

Biographical Information

Dr. Thomas was born October 25, 1934, in Lookout, West Virginia. He received the B. S. in Physics at New Mexico State University in 1957 and the Ph.D. in Physics at the University of Pittsburgh in 1963. He received a one year post-doctoral appointment at Service d'Aeronomie, C.N.R.S., Paris, France 1962-63. He served as 1st Lt., Sig. C., U. S. Army Electronics Laboratory, Ft. Monmouth, New Jersey, 1963-64. He was a Research Scientist at Los Alamos Scientific Laboratory, Los Alamos, New Mexico, in 1964; a member of the technical staff, Space Physics Laboratory, Aerospace Corporation, El Segundo, California from 1965-67; Assistant Professor (1967-69) and Associate Professor (1969-present), Astro-Geophysics Department, University of Colorado, Boulder, Colorado.

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Announcement of Lecture

EFFECTS OF DISTURBANCE ELECTRIC CURRENT ON THE UPPER ATMOSPHERE

Speaker: Dr. Keith D. Cole  
Time: 10:30 A.M., Tuesday, May 26, 1970  
Place: Research Institute, Room M-50  
The University of Alabama in Huntsville  
Huntsville, Alabama

Abstract

Some likely sources of electric currents and fields in the auroral ionosphere are reviewed. The dissipation of these currents and fields is then developed. Among other things, the following two major effects are shown to occur: (i) Increases of scale height and temperatures at the site of the auroral electrojet at altitudes above about 100 km. and (ii) Generation of high wind speeds in the auroral atmosphere.

Biographical Information

Dr. Keith D. Cole was born in Australia and received the Doctor of Science degree from the University of Queensland. In 1956 he was a member of the Antarctic Expedition to Macquarie Island. From 1957-63 he was Theoretical Physicist, Australia National Antarctic Research Expeditions. He has held the position of Research Associate with both the University of Chicago and the University of Colorado. Dr. Cole is presently a NAS-NRC Senior Post-Doctoral Fellow at Goddard Space Flight Center, Greenbelt, Maryland, on leave from La Trobe University, Victoria, Australia, where he is Foundation Professor of Physics. He has published over 50 papers in international journals, most on theory of magnetic disturbance.

\* \* \* \*

There is no charge for admission to the lecture and no pre-registration is required. For additional information, please call Dr. S. T. Wu, Associate Professor of Engineering, The University of Alabama in Huntsville, Telephone No. 895-6177.

RESEARCH INSTITUTE  
THE UNIVERSITY OF ALABAMA IN HUNTSVILLE

Announcement of lecture

THE FLARE FORECASTING FIASCO - IS EVERYBODY LOOKING AT THE SAME SUN?

Speaker: Dr. Frederick W. Ward, Jr.  
Time: 10:30 A.M., Tuesday, May 19, 1970  
Place: Science & Engineering Building, Room 127  
The University of Alabama in Huntsville  
Huntsville, Alabama

Abstract

The passionate dream of the oppressed solar flare forecaster is the scientific discovery of the cause, the energy source and the observable antecedent conditions for the solar flare. This "tunnel vision" has had a profound and deleterious effect on the state-of-the-art. It has diverted attention from many promising approaches and relegated them to limbo. More importantly, it has actually inhibited the scientific search for the underlying physical mechanisms. Examples of neglected aspects of the problem cover the entire range from observing techniques and equipment to the availability and accuracy of the archived scientific data. A cursory inspection of this data leads to an obvious conclusion - there are at least two suns up there.

Biographical Information

Dr. Frederick W. Ward, Jr., of the Space Physics Laboratory, Air Force Cambridge Research Laboratories, Bedford, Massachusetts, received the B.S., M.S., and Ph.D. degrees in meteorology from M.I.T. He directed the AFCRL Space Forecasting Program from 1966-69. His main research is on the problem of the general circulation of the solar atmosphere and the maintenance of the differential rotation.

## APPENDIX B

### List of Publications Resulting from this Contract

- (1) "Propagation of Hydromagnetic Waves in the Upper F-2 Region," Planet. Space Sci., Vol. 21, 1973.
- (2) "Kinetic Theory Analysis of Solar Wind Interaction With Planetary Objects," in Photon and Particle Interaction With Surfaces in Space (Ed. by R. J. L. Garard), D. Reidel Publishing Company, Dordrecht-Holland, 1973.
- (3) "The Dynamical Responses of the Thermosphere Due to a Geomagnetic Storm," AIAA Paper No. 74-217.
- (4) "An Analysis of the Upper Atmosphere Wind Observed by LOGACS," Planet. Space Sci., 1974.
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